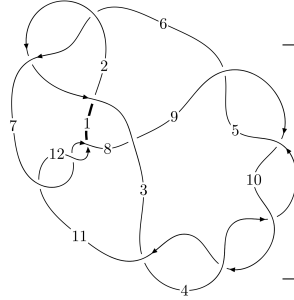
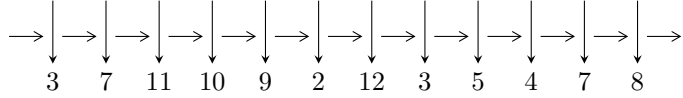


12n<sub>0581</sub> (K12n<sub>0581</sub>)



A knot diagram<sup>1</sup>

**Linearized knot diagram**



**Solving Sequence**

$$7, 11 \xrightarrow{c_{11}} 12 \xrightarrow{c_7} 4, 8 \xrightarrow{c_3} 3 \xrightarrow{c_2} 2 \xrightarrow{c_1} 1 \xrightarrow{c_6} 6 \xrightarrow{c_{10}} 10 \xrightarrow{c_4} 5 \xrightarrow{c_9} 9 \rightsquigarrow c_5, c_8, c_{12}$$

**Ideals for irreducible components<sup>2</sup> of  $X_{\text{par}}$**

$$I_1^u = \langle u^9 - u^8 - 7u^7 + 6u^6 + 14u^5 - 7u^4 - 5u^3 - 10u^2 + 4b + u,$$

$$- u^9 + u^8 + 7u^7 - 6u^6 - 14u^5 + 7u^4 + 5u^3 + 10u^2 + 4a - u - 4,$$

$$u^{10} - u^9 - 8u^8 + 7u^7 + 21u^6 - 13u^5 - 19u^4 + u^3 + 6u^2 - 2u - 1 \rangle$$

$$I_2^u = \langle 2u^7 + u^6 - 6u^5 - 6u^4 + 5u^3 + 11u^2 + 2b - 3u - 10,$$

$$- 5u^7 - 3u^6 + 16u^5 + 18u^4 - 15u^3 - 37u^2 + 8a + 9u + 39, u^8 - u^7 - 4u^6 + 2u^5 + 7u^4 + u^3 - 9u^2 - 3u + 8 \rangle$$

$$I_3^u = \langle b + a + 1, a^2 + 2a + 4, u + 1 \rangle$$

$$I_4^u = \langle b, a + 1, u + 1 \rangle$$

$$I_5^u = \langle b + a + 1, a^2 + 2a + 2, u - 1 \rangle$$

\* 5 irreducible components of  $\dim_{\mathbb{C}} = 0$ , with total 23 representations.

<sup>1</sup>The image of knot diagram is generated by the software “**Draw programme**” developed by Andrew Bartholomew(<http://www.layer8.co.uk/maths/draw/index.htm#Running-draw>), where we modified some parts for our purpose(<https://github.com/CATsTAILs/LinksPainter>).

<sup>2</sup>All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

I.

$$I_1^u = \langle u^9 - u^8 + \cdots + 4b + u, -u^9 + u^8 + \cdots + 4a - 4, u^{10} - u^9 + \cdots - 2u - 1 \rangle$$

(i) Arc colorings

$$a_7 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} 1 \\ u^2 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} \frac{1}{4}u^9 - \frac{1}{4}u^8 + \cdots + \frac{1}{4}u + 1 \\ -\frac{1}{4}u^9 + \frac{1}{4}u^8 + \cdots + \frac{5}{2}u^2 - \frac{1}{4}u \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -u \\ -u^3 + u \end{pmatrix}$$

$$a_3 = \begin{pmatrix} 1 \\ -\frac{1}{4}u^9 + \frac{1}{4}u^8 + \cdots + \frac{5}{2}u^2 - \frac{1}{4}u \end{pmatrix}$$

$$a_2 = \begin{pmatrix} 1 \\ -\frac{1}{4}u^9 + \frac{1}{4}u^8 + \cdots + \frac{3}{2}u^2 - \frac{1}{4}u \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -u^2 + 1 \\ -u^4 + 2u^2 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} u \\ -\frac{1}{4}u^8 + \frac{1}{4}u^7 + \cdots + \frac{1}{2}u - \frac{1}{4} \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} -\frac{1}{4}u^9 + \frac{1}{2}u^8 + \cdots - \frac{1}{4}u + \frac{5}{4} \\ \frac{1}{2}u^9 - \frac{3}{4}u^8 + \cdots + \frac{1}{2}u - \frac{1}{4} \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -\frac{1}{4}u^9 + \frac{1}{2}u^8 + \cdots + \frac{1}{4}u + \frac{3}{4} \\ \frac{1}{2}u^7 - \frac{7}{2}u^5 + \cdots - \frac{1}{2}u + \frac{1}{2} \end{pmatrix}$$

$$a_9 = \begin{pmatrix} \frac{1}{4}u^8 - \frac{1}{4}u^7 + \cdots - \frac{3}{2}u + \frac{1}{4} \\ -\frac{1}{4}u^9 + \frac{1}{4}u^8 + \cdots + \frac{7}{4}u + \frac{1}{2} \end{pmatrix}$$

(ii) Obstruction class = -1

$$(iii) \text{ Cusp Shapes} = 2u^9 - \frac{7}{2}u^8 - \frac{31}{2}u^7 + \frac{53}{2}u^6 + 38u^5 - 60u^4 - \frac{61}{2}u^3 + \frac{63}{2}u^2 + 14u - \frac{37}{2}$$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1$	$u^{10} + 17u^9 + \dots + 16u + 1$
$c_2, c_6, c_7$ $c_{11}, c_{12}$	$u^{10} - u^9 - 8u^8 + 7u^7 + 21u^6 - 13u^5 - 19u^4 + u^3 + 6u^2 - 2u - 1$
$c_3, c_4, c_5$ $c_9, c_{10}$	$u^{10} + 3u^9 + \dots + 12u + 2$
$c_8$	$u^{10} + 3u^9 + \dots + 108u + 58$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1$	$y^{10} - 49y^9 + \dots - 100y + 1$
$c_2, c_6, c_7$ $c_{11}, c_{12}$	$y^{10} - 17y^9 + \dots - 16y + 1$
$c_3, c_4, c_5$ $c_9, c_{10}$	$y^{10} + 13y^9 + \dots - 36y + 4$
$c_8$	$y^{10} - 51y^9 + \dots - 9460y + 3364$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.624283 + 0.413630I$ $a = 0.99995 + 1.69388I$ $b = 0.00005 - 1.69388I$	$11.16200 + 1.46971I$	$-6.43725 - 4.71631I$
$u = -0.624283 - 0.413630I$ $a = 0.99995 - 1.69388I$ $b = 0.00005 + 1.69388I$	$11.16200 - 1.46971I$	$-6.43725 + 4.71631I$
$u = 0.425863 + 0.318105I$ $a = 0.919105 - 0.860258I$ $b = 0.080895 + 0.860258I$	$2.03579 - 1.27062I$	$-6.43196 + 5.78765I$
$u = 0.425863 - 0.318105I$ $a = 0.919105 + 0.860258I$ $b = 0.080895 - 0.860258I$	$2.03579 + 1.27062I$	$-6.43196 - 5.78765I$
$u = -0.299023$ $a = 0.714196$ $b = 0.285804$	$-0.505065$	$-19.6030$
$u = 1.74982 + 0.35246I$ $a = 0.79690 - 1.70108I$ $b = 0.20310 + 1.70108I$	$-4.80759 - 8.37238I$	$-11.74586 + 3.25359I$
$u = 1.74982 - 0.35246I$ $a = 0.79690 + 1.70108I$ $b = 0.20310 - 1.70108I$	$-4.80759 + 8.37238I$	$-11.74586 - 3.25359I$
$u = -1.85465 + 0.19086I$ $a = 0.362420 + 0.933540I$ $b = 0.637580 - 0.933540I$	$-13.8130 + 4.9888I$	$-13.52702 - 3.37301I$
$u = -1.85465 - 0.19086I$ $a = 0.362420 - 0.933540I$ $b = 0.637580 + 0.933540I$	$-13.8130 - 4.9888I$	$-13.52702 + 3.37301I$
$u = 1.90554$ $a = 0.129056$ $b = 0.870944$	$-16.6134$	$-16.1130$

$$\text{II. } I_2^u = \langle 2u^7 + u^6 + \cdots + 2b - 10, -5u^7 - 3u^6 + \cdots + 8a + 39, u^8 - u^7 - 4u^6 + 2u^5 + 7u^4 + u^3 - 9u^2 - 3u + 8 \rangle$$

(i) Arc colorings

$$a_7 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} 1 \\ u^2 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} \frac{5}{8}u^7 + \frac{3}{8}u^6 + \cdots - \frac{9}{8}u - \frac{39}{8} \\ -u^7 - \frac{1}{2}u^6 + \cdots + \frac{3}{2}u + 5 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -u \\ -u^3 + u \end{pmatrix}$$

$$a_3 = \begin{pmatrix} -\frac{3}{8}u^7 - \frac{1}{8}u^6 + \cdots + \frac{3}{8}u + \frac{1}{8} \\ -u^7 - \frac{1}{2}u^6 + \cdots + \frac{3}{2}u + 5 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} -\frac{3}{8}u^7 - \frac{1}{8}u^6 + \cdots + \frac{3}{8}u + \frac{1}{8} \\ 1 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -u^2 + 1 \\ -u^4 + 2u^2 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} \frac{3}{8}u^7 + \frac{5}{8}u^6 + \cdots + \frac{9}{8}u - \frac{21}{8} \\ -\frac{1}{2}u^7 - \frac{1}{2}u^6 + \cdots - 3u^2 + 3 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} -\frac{1}{8}u^7 - \frac{3}{8}u^6 + \cdots + \frac{5}{8}u + \frac{7}{8} \\ -\frac{1}{2}u^7 + \frac{3}{2}u^5 + \cdots + \frac{1}{2}u + 3 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} \frac{1}{4}u^7 - \frac{1}{4}u^6 + \cdots - \frac{5}{4}u - \frac{3}{4} \\ u^7 + u^6 - 3u^5 - 4u^4 + 2u^3 + 6u^2 - u - 6 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} \frac{1}{8}u^7 - \frac{1}{8}u^6 + \cdots - \frac{9}{8}u - \frac{3}{8} \\ \frac{1}{2}u^7 + \frac{1}{2}u^6 + \cdots + 3u^2 - 3 \end{pmatrix}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes =  $2u^7 - 6u^5 - 4u^4 + 8u^3 + 10u^2 - 6u - 22$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1$	$u^8 + 9u^7 + 34u^6 + 76u^5 + 127u^4 + 179u^3 + 199u^2 + 153u + 64$
$c_2, c_6, c_7$ $c_{11}, c_{12}$	$u^8 - u^7 - 4u^6 + 2u^5 + 7u^4 + u^3 - 9u^2 - 3u + 8$
$c_3, c_4, c_5$ $c_9, c_{10}$	$(u^4 - u^3 + 3u^2 - 2u + 1)^2$
$c_8$	$(u^4 - u^3 + u^2 + 1)^2$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1$	$y^8 - 13y^7 + \dots + 2063y + 4096$
$c_2, c_6, c_7$ $c_{11}, c_{12}$	$y^8 - 9y^7 + 34y^6 - 76y^5 + 127y^4 - 179y^3 + 199y^2 - 153y + 64$
$c_3, c_4, c_5$ $c_9, c_{10}$	$(y^4 + 5y^3 + 7y^2 + 2y + 1)^2$
$c_8$	$(y^4 + y^3 + 3y^2 + 2y + 1)^2$



(vi) Complex Volumes and Cusp Shapes

Solutions to $I_2^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.974589 + 0.525375I$ $a = -0.575851 + 1.230320I$ $b = -0.395123 - 0.506844I$	$-3.50087 - 1.41510I$	$-13.8267 + 4.9087I$
$u = 0.974589 - 0.525375I$ $a = -0.575851 - 1.230320I$ $b = -0.395123 + 0.506844I$	$-3.50087 + 1.41510I$	$-13.8267 - 4.9087I$
$u = -0.728625 + 0.959908I$ $a = -0.72016 - 2.57269I$ $b = -0.10488 + 1.55249I$	$3.50087 + 3.16396I$	$-10.17326 - 2.56480I$
$u = -0.728625 - 0.959908I$ $a = -0.72016 + 2.57269I$ $b = -0.10488 - 1.55249I$	$3.50087 - 3.16396I$	$-10.17326 + 2.56480I$
$u = -1.326400 + 0.194967I$ $a = -0.267111 + 0.013410I$ $b = -0.395123 - 0.506844I$	$-3.50087 - 1.41510I$	$-13.8267 + 4.9087I$
$u = -1.326400 - 0.194967I$ $a = -0.267111 - 0.013410I$ $b = -0.395123 + 0.506844I$	$-3.50087 + 1.41510I$	$-13.8267 - 4.9087I$
$u = 1.58043 + 0.04862I$ $a = -0.374382 + 0.959864I$ $b = -0.10488 - 1.55249I$	$3.50087 - 3.16396I$	$-10.17326 + 2.56480I$
$u = 1.58043 - 0.04862I$ $a = -0.374382 - 0.959864I$ $b = -0.10488 + 1.55249I$	$3.50087 + 3.16396I$	$-10.17326 - 2.56480I$

$$\text{III. } I_3^u = \langle b + a + 1, a^2 + 2a + 4, u + 1 \rangle$$

(i) Arc colorings

$$a_7 = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} a \\ -a - 1 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} -1 \\ -a - 1 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} -1 \\ -a \end{pmatrix}$$

$$a_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -1 \\ -a - 1 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} -a - 3 \\ 3 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -a + 1 \\ 2a + 2 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} a + 2 \\ -3 \end{pmatrix}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes = -12

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1, c_2, c_7$	$(u - 1)^2$
$c_3, c_4, c_5$ $c_8, c_9, c_{10}$	$u^2 + 3$
$c_6, c_{11}, c_{12}$	$(u + 1)^2$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1, c_2, c_6$ $c_7, c_{11}, c_{12}$	$(y - 1)^2$
$c_3, c_4, c_5$ $c_8, c_9, c_{10}$	$(y + 3)^2$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_3^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -1.00000$ $a = -1.00000 + 1.73205I$ $b = -1.73205I$	9.86960	-12.0000
$u = -1.00000$ $a = -1.00000 - 1.73205I$ $b = 1.73205I$	9.86960	-12.0000

$$\text{IV. } I_4^u = \langle b, a + 1, u + 1 \rangle$$

(i) Arc colorings

$$a_7 = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes = -12

(iv) **u**-Polynomials at the component

Crossings	<b>u</b> -Polynomials at each crossing
$c_1, c_2, c_7$	$u - 1$
$c_3, c_4, c_5$ $c_8, c_9, c_{10}$	$u$
$c_6, c_{11}, c_{12}$	$u + 1$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1, c_2, c_6$ $c_7, c_{11}, c_{12}$	$y - 1$
$c_3, c_4, c_5$ $c_8, c_9, c_{10}$	$y$



(vi) Complex Volumes and Cusp Shapes

Solutions to $I_4^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -1.00000$		
$a = -1.00000$	-3.28987	-12.0000
$b = 0$		

$$\mathbf{V}. I_5^u = \langle b + a + 1, a^2 + 2a + 2, u - 1 \rangle$$

**(i) Arc colorings**

$$a_7 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} a \\ -a - 1 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} -1 \\ -a - 1 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} -1 \\ -a \end{pmatrix}$$

$$a_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} 1 \\ a + 1 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} -a - 1 \\ 1 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} a + 1 \\ 0 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -a - 2 \\ 1 \end{pmatrix}$$

**(ii) Obstruction class = 1**

**(iii) Cusp Shapes = -12**

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1, c_6, c_{11}$ $c_{12}$	$(u - 1)^2$
$c_2, c_7$	$(u + 1)^2$
$c_3, c_4, c_5$ $c_8, c_9, c_{10}$	$u^2 + 1$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1, c_2, c_6$ $c_7, c_{11}, c_{12}$	$(y - 1)^2$
$c_3, c_4, c_5$ $c_8, c_9, c_{10}$	$(y + 1)^2$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_5^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 1.00000$ $a = -1.00000 + 1.00000I$ $b = -1.000000I$	0	-12.0000
$u = 1.00000$ $a = -1.00000 - 1.00000I$ $b = 1.000000I$	0	-12.0000

## VI. u-Polynomials

Crossings	u-Polynomials at each crossing
$c_1$	$(u-1)^5$ $\cdot (u^8 + 9u^7 + 34u^6 + 76u^5 + 127u^4 + 179u^3 + 199u^2 + 153u + 64)$ $\cdot (u^{10} + 17u^9 + \dots + 16u + 1)$
$c_2, c_7$	$(u-1)^3(u+1)^2(u^8 - u^7 - 4u^6 + 2u^5 + 7u^4 + u^3 - 9u^2 - 3u + 8)$ $\cdot (u^{10} - u^9 - 8u^8 + 7u^7 + 21u^6 - 13u^5 - 19u^4 + u^3 + 6u^2 - 2u - 1)$
$c_3, c_4, c_5$ $c_9, c_{10}$	$u(u^2 + 1)(u^2 + 3)(u^4 - u^3 + \dots - 2u + 1)^2(u^{10} + 3u^9 + \dots + 12u + 2)$
$c_6, c_{11}, c_{12}$	$(u-1)^2(u+1)^3(u^8 - u^7 - 4u^6 + 2u^5 + 7u^4 + u^3 - 9u^2 - 3u + 8)$ $\cdot (u^{10} - u^9 - 8u^8 + 7u^7 + 21u^6 - 13u^5 - 19u^4 + u^3 + 6u^2 - 2u - 1)$
$c_8$	$u(u^2 + 1)(u^2 + 3)(u^4 - u^3 + u^2 + 1)^2(u^{10} + 3u^9 + \dots + 108u + 58)$

## VII. Riley Polynomials

Crossings	Riley Polynomials at each crossing
$c_1$	$((y-1)^5)(y^8 - 13y^7 + \dots + 2063y + 4096)$ $\cdot (y^{10} - 49y^9 + \dots - 100y + 1)$
$c_2, c_6, c_7$ $c_{11}, c_{12}$	$(y-1)^5$ $\cdot (y^8 - 9y^7 + 34y^6 - 76y^5 + 127y^4 - 179y^3 + 199y^2 - 153y + 64)$ $\cdot (y^{10} - 17y^9 + \dots - 16y + 1)$
$c_3, c_4, c_5$ $c_9, c_{10}$	$y(y+1)^2(y+3)^2(y^4 + 5y^3 + 7y^2 + 2y + 1)^2$ $\cdot (y^{10} + 13y^9 + \dots - 36y + 4)$
$c_8$	$y(y+1)^2(y+3)^2(y^4 + y^3 + 3y^2 + 2y + 1)^2$ $\cdot (y^{10} - 51y^9 + \dots - 9460y + 3364)$