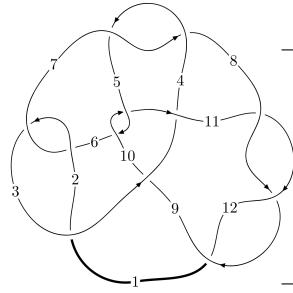
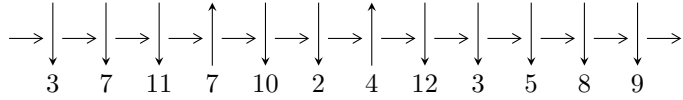


12n<sub>0590</sub> (K12n<sub>0590</sub>)



A knot diagram<sup>1</sup>

**Linearized knot diagram**



**Solving Sequence**

$$8, 11 \xrightarrow{c_{11}} 12 \xrightarrow{c_8} 9 \xrightarrow{c_{12}} 1, 4 \xrightarrow{c_3} 3 \xrightarrow{c_7} 7 \xrightarrow{c_4} 5 \xrightarrow{c_2} 2 \xrightarrow{c_6} 6 \xrightarrow{c_{10}} 10 \rightsquigarrow c_1, c_5, c_9$$

**Ideals for irreducible components<sup>2</sup> of  $X_{\text{par}}$**

$$I_1^u = \langle 24558310627155u^{21} + 76226351773367u^{20} + \dots + 826989943797623b - 264882167921191, \\ - 1.11442 \times 10^{15}u^{21} - 1.96283 \times 10^{15}u^{20} + \dots + 5.78893 \times 10^{15}a + 1.82405 \times 10^{14}, u^{22} + u^{21} + \dots - 12u \rangle$$

$$I_2^u = \langle -u^7 + 5u^5 + u^4 - 7u^3 - 2u^2 + b + 2u, u^{10} - 8u^8 + 23u^6 - u^5 - 29u^4 + 4u^3 + 15u^2 + a - 4u - 1, \\ u^{11} - 8u^9 - u^8 + 23u^7 + 5u^6 - 28u^5 - 7u^4 + 13u^3 + 2u^2 - 2u - 1 \rangle$$

\* 2 irreducible components of  $\dim_{\mathbb{C}} = 0$ , with total 33 representations.

<sup>1</sup>The image of knot diagram is generated by the software “**Draw programme**” developed by Andrew Bartholomew(<http://www.layer8.co.uk/maths/draw/index.htm#Running-draw>), where we modified some parts for our purpose(<https://github.com/CATsTAILs/LinksPainter>).

<sup>2</sup>All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

I.

$$I_1^u = \langle 2.46 \times 10^{13} u^{21} + 7.62 \times 10^{13} u^{20} + \dots + 8.27 \times 10^{14} b - 2.65 \times 10^{14}, -1.11 \times 10^{15} u^{21} - 1.96 \times 10^{15} u^{20} + \dots + 5.79 \times 10^{15} a + 1.82 \times 10^{14}, u^{22} + u^{21} + \dots - 12u + 7 \rangle$$

(i) Arc colorings

$$a_8 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} 1 \\ u^2 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -u \\ -u^3 + u \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -u^2 + 1 \\ -u^4 + 2u^2 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 0.192509u^{21} + 0.339065u^{20} + \dots + 1.66279u - 0.0315092 \\ -0.0296960u^{21} - 0.0921733u^{20} + \dots - 0.726566u + 0.320297 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} 0.162813u^{21} + 0.246892u^{20} + \dots + 0.936221u + 0.288788 \\ -0.0296960u^{21} - 0.0921733u^{20} + \dots - 0.726566u + 0.320297 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -0.308808u^{21} - 0.423908u^{20} + \dots - 4.90570u + 3.29659 \\ -0.0596766u^{21} - 0.0810654u^{20} + \dots + 1.56726u + 0.227107 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -0.193496u^{21} - 0.277468u^{20} + \dots + 1.30428u + 0.534875 \\ 0.117215u^{21} + 0.139319u^{20} + \dots - 0.863885u - 0.561596 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} -0.340275u^{21} - 0.520418u^{20} + \dots - 1.11925u + 4.17997 \\ -0.0351453u^{21} - 0.0475187u^{20} + \dots - 0.171350u - 0.289676 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} 0.285354u^{21} + 0.292486u^{20} + \dots - 3.40992u - 0.148449 \\ -0.0727430u^{21} - 0.200381u^{20} + \dots + 1.21818u + 0.857957 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} -0.115641u^{21} - 0.306721u^{20} + \dots - 4.96241u + 2.03078 \\ 0.148193u^{21} + 0.158198u^{20} + \dots + 1.75028u - 0.764596 \end{pmatrix}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes

$$= -\frac{5642994687059}{826989943797623} u^{21} + \frac{135118301910616}{826989943797623} u^{20} + \dots + \frac{9106726180669764}{826989943797623} u - \frac{9982429531246890}{826989943797623}$$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1$	$u^{22} + 44u^{21} + \dots + 111391u + 14641$
$c_2, c_6$	$u^{22} - 22u^{20} + \dots - 95u - 121$
$c_3$	$u^{22} + 3u^{21} + \dots - 13u - 5$
$c_4, c_7$	$u^{22} + 5u^{21} + \dots + 2u + 1$
$c_5, c_{10}$	$u^{22} - u^{21} + \dots - 123u - 173$
$c_8, c_{11}, c_{12}$	$u^{22} + u^{21} + \dots - 12u + 7$
$c_9$	$u^{22} - 6u^{21} + \dots - 66255u - 6691$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1$	$y^{22} - 140y^{21} + \dots - 15971954947y + 214358881$
$c_2, c_6$	$y^{22} - 44y^{21} + \dots - 111391y + 14641$
$c_3$	$y^{22} - 3y^{21} + \dots - 319y + 25$
$c_4, c_7$	$y^{22} + 21y^{21} + \dots - 40y + 1$
$c_5, c_{10}$	$y^{22} - 9y^{21} + \dots - 332065y + 29929$
$c_8, c_{11}, c_{12}$	$y^{22} - 37y^{21} + \dots - 186y + 49$
$c_9$	$y^{22} - 122y^{21} + \dots - 1381946559y + 44769481$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.813375 + 0.695520I$ $a = -0.621272 + 0.888438I$ $b = 0.930849 - 0.474906I$	$-3.57261 + 1.24203I$	$-13.51654 - 2.69011I$
$u = -0.813375 - 0.695520I$ $a = -0.621272 - 0.888438I$ $b = 0.930849 + 0.474906I$	$-3.57261 - 1.24203I$	$-13.51654 + 2.69011I$
$u = 1.14577$ $a = -1.21118$ $b = -0.786013$	$-5.50697$	$-18.3200$
$u = 0.689625 + 0.112216I$ $a = 0.60149 + 2.27758I$ $b = 0.783398 - 0.626729I$	$-2.94663 + 3.18256I$	$-14.1931 - 4.5682I$
$u = 0.689625 - 0.112216I$ $a = 0.60149 - 2.27758I$ $b = 0.783398 + 0.626729I$	$-2.94663 - 3.18256I$	$-14.1931 + 4.5682I$
$u = -1.42854 + 0.12438I$ $a = -0.118525 + 0.124764I$ $b = -0.515254 - 0.715646I$	$-3.21068 - 1.94100I$	$-8.44193 + 4.49457I$
$u = -1.42854 - 0.12438I$ $a = -0.118525 - 0.124764I$ $b = -0.515254 + 0.715646I$	$-3.21068 + 1.94100I$	$-8.44193 - 4.49457I$
$u = 0.439809 + 0.342493I$ $a = 0.658925 - 0.629517I$ $b = 0.104940 + 1.110730I$	$2.79807 - 1.24078I$	$-2.39806 + 5.78935I$
$u = 0.439809 - 0.342493I$ $a = 0.658925 + 0.629517I$ $b = 0.104940 - 1.110730I$	$2.79807 + 1.24078I$	$-2.39806 - 5.78935I$
$u = -1.47818 + 0.29462I$ $a = -0.196203 + 1.126150I$ $b = -0.95775 - 1.05246I$	$-10.23510 - 1.31699I$	$-13.69750 + 1.12150I$

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -1.47818 - 0.29462I$ $a = -0.196203 - 1.126150I$ $b = -0.95775 + 1.05246I$	$-10.23510 + 1.31699I$	$-13.69750 - 1.12150I$
$u = -0.389548$ $a = 0.489094$ $b = 0.423427$	$-0.651086$	$-15.1770$
$u = 0.055531 + 0.375870I$ $a = 3.27936 - 0.40059I$ $b = -0.735830 + 0.136693I$	$-1.05201 + 2.37866I$	$-7.37484 + 0.30199I$
$u = 0.055531 - 0.375870I$ $a = 3.27936 + 0.40059I$ $b = -0.735830 - 0.136693I$	$-1.05201 - 2.37866I$	$-7.37484 - 0.30199I$
$u = 1.54618 + 0.54222I$ $a = 0.045294 + 0.967197I$ $b = -1.17072 - 0.88115I$	$-11.07800 - 6.11809I$	$-13.7689 + 3.7652I$
$u = 1.54618 - 0.54222I$ $a = 0.045294 - 0.967197I$ $b = -1.17072 + 0.88115I$	$-11.07800 + 6.11809I$	$-13.7689 - 3.7652I$
$u = -1.78595$ $a = 1.55694$ $b = 0.706857$	$-16.3042$	$-23.6750$
$u = 1.92376 + 0.13131I$ $a = -0.017132 + 0.779055I$ $b = 1.13005 - 1.28185I$	$16.3680 - 1.4414I$	$-13.10276 + 0.15786I$
$u = 1.92376 - 0.13131I$ $a = -0.017132 - 0.779055I$ $b = 1.13005 + 1.28185I$	$16.3680 + 1.4414I$	$-13.10276 - 0.15786I$
$u = -1.92464 + 0.18067I$ $a = 0.224483 + 0.973602I$ $b = 1.24119 - 1.08462I$	$15.7619 + 10.2221I$	$-12.91781 - 4.03002I$

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -1.92464 - 0.18067I$	$15.7619 - 10.2221I$	$-12.91781 + 4.03002I$
$a = 0.224483 - 0.973602I$		
$b = 1.24119 + 1.08462I$		
$u = 2.00938$	$-17.7473$	$-16.0060$
$a = -0.119138$		
$b = 1.03398$		

$$\text{II. } I_2^u = \langle -u^7 + 5u^5 + u^4 - 7u^3 - 2u^2 + b + 2u, u^{10} - 8u^8 + \dots + a - 1, u^{11} - 8u^9 + \dots - 2u - 1 \rangle$$

(i) Arc colorings

$$a_8 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} 1 \\ u^2 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -u \\ -u^3 + u \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -u^2 + 1 \\ -u^4 + 2u^2 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} -u^{10} + 8u^8 - 23u^6 + u^5 + 29u^4 - 4u^3 - 15u^2 + 4u + 1 \\ u^7 - 5u^5 - u^4 + 7u^3 + 2u^2 - 2u \end{pmatrix}$$

$$a_3 = \begin{pmatrix} -u^{10} + 8u^8 + u^7 - 23u^6 - 4u^5 + 28u^4 + 3u^3 - 13u^2 + 2u + 1 \\ u^7 - 5u^5 - u^4 + 7u^3 + 2u^2 - 2u \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -u^8 - u^7 + 6u^6 + 6u^5 - 11u^4 - 10u^3 + 7u^2 + 4u - 3 \\ -u^{10} + 7u^8 + u^7 - 17u^6 - 4u^5 + 17u^4 + 4u^3 - 7u^2 + 1 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} u^{10} - 8u^8 - u^7 + 23u^6 + 5u^5 - 28u^4 - 6u^3 + 14u^2 - 3 \\ -u^{10} + 7u^8 + u^7 - 17u^6 - 4u^5 + 16u^4 + 4u^3 - 4u^2 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} u^8 + u^7 - 6u^6 - 5u^5 + 11u^4 + 7u^3 - 7u^2 - 2u + 2 \\ u^7 - 5u^5 - 2u^4 + 7u^3 + 5u^2 - 2u - 1 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} u^{10} - 7u^8 - u^7 + 18u^6 + 5u^5 - 20u^4 - 7u^3 + 9u^2 + 2u - 2 \\ -u^7 + 5u^5 + u^4 - 7u^3 - 3u^2 + 2u + 1 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} u^9 + u^8 - 6u^7 - 6u^6 + 11u^5 + 10u^4 - 7u^3 - 3u^2 + 3u \\ -u^3 + 2u + 1 \end{pmatrix}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes

$$= 2u^{10} - 2u^9 - 17u^8 + 12u^7 + 54u^6 - 24u^5 - 73u^4 + 21u^3 + 35u^2 - 12u - 17$$



(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1$	$u^{11} - 11u^{10} + \dots + 5u - 1$
$c_2$	$u^{11} - 3u^{10} - u^9 + 8u^8 - 2u^7 - 10u^6 + 5u^5 + 6u^4 - 5u^3 - 2u^2 + 3u - 1$
$c_3$	$u^{11} - 2u^{10} + 3u^9 - 2u^8 + u^7 + u^6 - u^5 + u^4 + 5u^3 - 4u^2 - u + 1$
$c_4$	$u^{11} + 3u^9 - 6u^8 + u^7 - 20u^6 - 4u^5 - 26u^4 - 6u^3 - 15u^2 - 4u - 3$
$c_5$	$u^{11} + 4u^9 - u^8 + 4u^7 - 4u^6 - u^5 - 5u^4 - 2u^3 - 3u^2 - u - 1$
$c_6$	$u^{11} + 3u^{10} - u^9 - 8u^8 - 2u^7 + 10u^6 + 5u^5 - 6u^4 - 5u^3 + 2u^2 + 3u + 1$
$c_7$	$u^{11} + 3u^9 + 6u^8 + u^7 + 20u^6 - 4u^5 + 26u^4 - 6u^3 + 15u^2 - 4u + 3$
$c_8$	$u^{11} - 8u^9 + u^8 + 23u^7 - 5u^6 - 28u^5 + 7u^4 + 13u^3 - 2u^2 - 2u + 1$
$c_9$	$u^{11} - 5u^{10} + \dots + 5u - 1$
$c_{10}$	$u^{11} + 4u^9 + u^8 + 4u^7 + 4u^6 - u^5 + 5u^4 - 2u^3 + 3u^2 - u + 1$
$c_{11}, c_{12}$	$u^{11} - 8u^9 - u^8 + 23u^7 + 5u^6 - 28u^5 - 7u^4 + 13u^3 + 2u^2 - 2u - 1$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1$	$y^{11} - 31y^{10} + \dots - 19y - 1$
$c_2, c_6$	$y^{11} - 11y^{10} + \dots + 5y - 1$
$c_3$	$y^{11} + 2y^{10} + \dots + 9y - 1$
$c_4, c_7$	$y^{11} + 6y^{10} + \dots - 74y - 9$
$c_5, c_{10}$	$y^{11} + 8y^{10} + \dots - 5y - 1$
$c_8, c_{11}, c_{12}$	$y^{11} - 16y^{10} + \dots + 8y - 1$
$c_9$	$y^{11} - 33y^{10} + \dots - 11y - 1$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_2^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -1.137090 + 0.296275I$ $a = -0.112692 - 0.807373I$ $b = -0.583202 - 0.143276I$	$-4.14701 - 1.11671I$	$-15.2828 + 0.9622I$
$u = -1.137090 - 0.296275I$ $a = -0.112692 + 0.807373I$ $b = -0.583202 + 0.143276I$	$-4.14701 + 1.11671I$	$-15.2828 - 0.9622I$
$u = 0.592958 + 0.265544I$ $a = -0.648705 - 0.517226I$ $b = 0.224636 + 1.262620I$	$2.05237 - 0.90366I$	$-12.99258 + 0.59002I$
$u = 0.592958 - 0.265544I$ $a = -0.648705 + 0.517226I$ $b = 0.224636 - 1.262620I$	$2.05237 + 0.90366I$	$-12.99258 - 0.59002I$
$u = 1.52480 + 0.07903I$ $a = -0.465943 + 0.973368I$ $b = -1.133880 - 0.701740I$	$-7.77644 - 4.37367I$	$-13.16320 + 3.47973I$
$u = 1.52480 - 0.07903I$ $a = -0.465943 - 0.973368I$ $b = -1.133880 + 0.701740I$	$-7.77644 + 4.37367I$	$-13.16320 - 3.47973I$
$u = -1.59014 + 0.09758I$ $a = 0.004921 - 0.710560I$ $b = -0.62219 + 1.27817I$	$-5.52612 + 2.32410I$	$-12.39685 - 2.31746I$
$u = -1.59014 - 0.09758I$ $a = 0.004921 + 0.710560I$ $b = -0.62219 - 1.27817I$	$-5.52612 - 2.32410I$	$-12.39685 + 2.31746I$
$u = -0.300210 + 0.263166I$ $a = -1.31732 + 2.93205I$ $b = 0.867895 - 0.444411I$	$-1.38510 + 3.17083I$	$-10.45196 - 6.81024I$
$u = -0.300210 - 0.263166I$ $a = -1.31732 - 2.93205I$ $b = 0.867895 + 0.444411I$	$-1.38510 - 3.17083I$	$-10.45196 + 6.81024I$

	Solutions to $I_2^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u =$	1.81936		
$a =$	1.07948	-15.7834	-7.42530
$b =$	0.493485		

### III. u-Polynomials

Crossings	u-Polynomials at each crossing
$c_1$	$(u^{11} - 11u^{10} + \dots + 5u - 1)(u^{22} + 44u^{21} + \dots + 111391u + 14641)$
$c_2$	$(u^{11} - 3u^{10} - u^9 + 8u^8 - 2u^7 - 10u^6 + 5u^5 + 6u^4 - 5u^3 - 2u^2 + 3u - 1)$ $\cdot (u^{22} - 22u^{20} + \dots - 95u - 121)$
$c_3$	$(u^{11} - 2u^{10} + 3u^9 - 2u^8 + u^7 + u^6 - u^5 + u^4 + 5u^3 - 4u^2 - u + 1)$ $\cdot (u^{22} + 3u^{21} + \dots - 13u - 5)$
$c_4$	$(u^{11} + 3u^9 - 6u^8 + u^7 - 20u^6 - 4u^5 - 26u^4 - 6u^3 - 15u^2 - 4u - 3)$ $\cdot (u^{22} + 5u^{21} + \dots + 2u + 1)$
$c_5$	$(u^{11} + 4u^9 - u^8 + 4u^7 - 4u^6 - u^5 - 5u^4 - 2u^3 - 3u^2 - u - 1)$ $\cdot (u^{22} - u^{21} + \dots - 123u - 173)$
$c_6$	$(u^{11} + 3u^{10} - u^9 - 8u^8 - 2u^7 + 10u^6 + 5u^5 - 6u^4 - 5u^3 + 2u^2 + 3u + 1)$ $\cdot (u^{22} - 22u^{20} + \dots - 95u - 121)$
$c_7$	$(u^{11} + 3u^9 + 6u^8 + u^7 + 20u^6 - 4u^5 + 26u^4 - 6u^3 + 15u^2 - 4u + 3)$ $\cdot (u^{22} + 5u^{21} + \dots + 2u + 1)$
$c_8$	$(u^{11} - 8u^9 + u^8 + 23u^7 - 5u^6 - 28u^5 + 7u^4 + 13u^3 - 2u^2 - 2u + 1)$ $\cdot (u^{22} + u^{21} + \dots - 12u + 7)$
$c_9$	$(u^{11} - 5u^{10} + \dots + 5u - 1)(u^{22} - 6u^{21} + \dots - 66255u - 6691)$
$c_{10}$	$(u^{11} + 4u^9 + u^8 + 4u^7 + 4u^6 - u^5 + 5u^4 - 2u^3 + 3u^2 - u + 1)$ $\cdot (u^{22} - u^{21} + \dots - 123u - 173)$
$c_{11}, c_{12}$	$(u^{11} - 8u^9 - u^8 + 23u^7 + 5u^6 - 28u^5 - 7u^4 + 13u^3 + 2u^2 - 2u - 1)$ $\cdot (u^{22} + u^{21} + \dots - 12u + 7)$

#### IV. Riley Polynomials

Crossings	Riley Polynomials at each crossing
$c_1$	$(y^{11} - 31y^{10} + \dots - 19y - 1)$ $\cdot (y^{22} - 140y^{21} + \dots - 15971954947y + 214358881)$
$c_2, c_6$	$(y^{11} - 11y^{10} + \dots + 5y - 1)(y^{22} - 44y^{21} + \dots - 111391y + 14641)$
$c_3$	$(y^{11} + 2y^{10} + \dots + 9y - 1)(y^{22} - 3y^{21} + \dots - 319y + 25)$
$c_4, c_7$	$(y^{11} + 6y^{10} + \dots - 74y - 9)(y^{22} + 21y^{21} + \dots - 40y + 1)$
$c_5, c_{10}$	$(y^{11} + 8y^{10} + \dots - 5y - 1)(y^{22} - 9y^{21} + \dots - 332065y + 29929)$
$c_8, c_{11}, c_{12}$	$(y^{11} - 16y^{10} + \dots + 8y - 1)(y^{22} - 37y^{21} + \dots - 186y + 49)$
$c_9$	$(y^{11} - 33y^{10} + \dots - 11y - 1)$ $\cdot (y^{22} - 122y^{21} + \dots - 1381946559y + 44769481)$