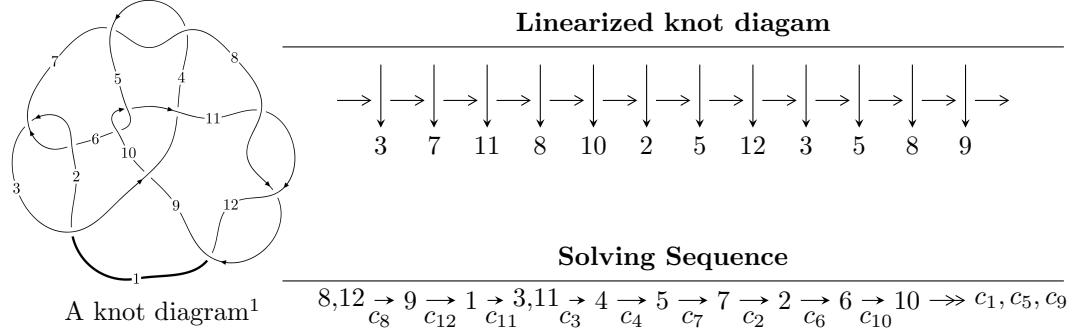


$12n_{0591}$  ( $K12n_{0591}$ )



**Ideals for irreducible components<sup>2</sup> of  $X_{\text{par}}$**

$$I_1^u = \langle -u^6 + 3u^5 + 3u^4 - 17u^3 + 12u^2 + b + 4u - 2, 2u^7 - 9u^6 + 2u^5 + 44u^4 - 70u^3 + 21u^2 + a + 16u - 4, u^8 - 4u^7 - u^6 + 22u^5 - 25u^4 - 4u^3 + 12u^2 + u - 1 \rangle$$

$$I_2^u = \langle u^4 + 2u^3 - u^2 + b - u, u^4 + 3u^3 + u^2 + a - 2u - 1, u^5 + 3u^4 - 3u^2 + u - 1 \rangle$$

\* 2 irreducible components of  $\dim_{\mathbb{C}} = 0$ , with total 13 representations.

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<sup>1</sup>The image of knot diagram is generated by the software “**Draw programme**” developed by Andrew Bartholomew(<http://www.layer8.co.uk/math/draw/index.htm#Running-draw>), where we modified some parts for our purpose(<https://github.com/CATsTAILs/LinksPainter>).

<sup>2</sup>All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

$$\text{I. } I_1^u = \langle -u^6 + 3u^5 + 3u^4 - 17u^3 + 12u^2 + b + 4u - 2, 2u^7 - 9u^6 + \dots + a - 4, u^8 - 4u^7 + \dots + u - 1 \rangle$$

(i) **Arc colorings**

$$\begin{aligned} a_8 &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ a_{12} &= \begin{pmatrix} 0 \\ u \end{pmatrix} \\ a_9 &= \begin{pmatrix} 1 \\ u^2 \end{pmatrix} \\ a_1 &= \begin{pmatrix} -u \\ -u^3 + u \end{pmatrix} \\ a_3 &= \begin{pmatrix} -2u^7 + 9u^6 - 2u^5 - 44u^4 + 70u^3 - 21u^2 - 16u + 4 \\ u^6 - 3u^5 - 3u^4 + 17u^3 - 12u^2 - 4u + 2 \end{pmatrix} \\ a_{11} &= \begin{pmatrix} u \\ u \end{pmatrix} \\ a_4 &= \begin{pmatrix} -3u^7 + 12u^6 + u^5 - 61u^4 + 82u^3 - 17u^2 - 18u + 4 \\ -u^7 + 4u^6 - 20u^4 + 29u^3 - 8u^2 - 6u + 2 \end{pmatrix} \\ a_5 &= \begin{pmatrix} -2u^7 + 8u^6 + u^5 - 41u^4 + 53u^3 - 9u^2 - 12u + 2 \\ -u^7 + 4u^6 - 20u^4 + 29u^3 - 8u^2 - 6u + 2 \end{pmatrix} \\ a_7 &= \begin{pmatrix} -u^6 + 2u^5 + 4u^4 - 11u^3 + 6u^2 + 1 \\ -u^7 + u^6 + 7u^5 - 8u^4 - 10u^3 + 12u^2 + u - 1 \end{pmatrix} \\ a_2 &= \begin{pmatrix} u^7 - u^6 - 8u^5 + 9u^4 + 15u^3 - 19u^2 - u + 1 \\ 3u^7 - 6u^6 - 16u^5 + 37u^4 + u^3 - 25u^2 + 2 \end{pmatrix} \\ a_6 &= \begin{pmatrix} 3u^7 - 11u^6 - 6u^5 + 58u^4 - 56u^3 - u^2 + 12u - 1 \\ 2u^7 - 8u^6 - 5u^5 + 42u^4 - 35u^3 - 4u^2 + 6u - 1 \end{pmatrix} \\ a_{10} &= \begin{pmatrix} -u^7 + 2u^6 + 5u^5 - 12u^4 + u^3 + 6u^2 + u \\ -2u^7 + 4u^6 + 10u^5 - 24u^4 + 2u^3 + 13u^2 + u - 1 \end{pmatrix} \end{aligned}$$

(ii) **Obstruction class** = -1

(iii) **Cusp Shapes** =  $-8u^7 + 32u^6 + 4u^5 - 163u^4 + 213u^3 - 40u^2 - 51u - 4$

**(iv) u-Polynomials at the component**

Crossings	u-Polynomials at each crossing
$c_1$	$u^8 + 7u^7 + 35u^6 + 244u^5 + 493u^4 + 885u^3 + 772u^2 + 608u + 64$
$c_2, c_6$	$u^8 - 9u^7 + 37u^6 - 88u^5 + 125u^4 - 101u^3 + 34u^2 + 8u - 8$
$c_3$	$u^8 + 2u^7 + 4u^6 - 6u^5 + 10u^4 - 3u^3 + 5u^2 - u - 1$
$c_4, c_7$	$u^8 - 3u^7 + 9u^6 - 2u^5 - 14u^3 - 19u^2 - 8u - 1$
$c_5, c_9, c_{10}$	$u^8 + 2u^7 + 10u^6 + 51u^5 + 8u^4 + 28u^3 + 10u^2 + 2u + 1$
$c_8, c_{11}, c_{12}$	$u^8 + 4u^7 - u^6 - 22u^5 - 25u^4 + 4u^3 + 12u^2 - u - 1$

**(v) Riley Polynomials at the component**

Crossings	Riley Polynomials at each crossing
$c_1$	$y^8 + 21y^7 + \dots - 270848y + 4096$
$c_2, c_6$	$y^8 - 7y^7 + 35y^6 - 244y^5 + 493y^4 - 885y^3 + 772y^2 - 608y + 64$
$c_3$	$y^8 + 4y^7 + 60y^6 + 66y^5 + 106y^4 + 71y^3 - y^2 - 11y + 1$
$c_4, c_7$	$y^8 + 9y^7 + 69y^6 - 126y^5 - 448y^4 - 246y^3 + 137y^2 - 26y + 1$
$c_5, c_9, c_{10}$	$y^8 + 16y^7 - 88y^6 - 2533y^5 - 2598y^4 - 808y^3 + 4y^2 + 16y + 1$
$c_8, c_{11}, c_{12}$	$y^8 - 18y^7 + 127y^6 - 442y^5 + 783y^4 - 658y^3 + 202y^2 - 25y + 1$

**(vi) Complex Volumes and Cusp Shapes**

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 1.357740 + 0.195354I$		
$a = -0.300439 + 0.153458I$	$-3.40670 + 1.82723I$	$-16.1033 - 2.4545I$
$b = -0.766915 + 0.837154I$		
$u = 1.357740 - 0.195354I$		
$a = -0.300439 - 0.153458I$	$-3.40670 - 1.82723I$	$-16.1033 + 2.4545I$
$b = -0.766915 - 0.837154I$		
$u = -0.432193 + 0.048912I$		
$a = 0.26933 + 2.64283I$	$2.34111 - 2.75408I$	$-10.98687 + 7.43235I$
$b = 0.144443 + 0.793261I$		
$u = -0.432193 - 0.048912I$		
$a = 0.26933 - 2.64283I$	$2.34111 + 2.75408I$	$-10.98687 - 7.43235I$
$b = 0.144443 - 0.793261I$		
$u = 0.276903$		
$a = -0.812540$	$-0.562481$	$-17.6050$
$b = 0.311154$		
$u = 2.08809 + 0.20687I$		
$a = 0.80818 + 1.18869I$	$3.25293 - 6.36321I$	$-12.47350 + 2.40837I$
$b = 1.72548 + 2.10674I$		
$u = 2.08809 - 0.20687I$		
$a = 0.80818 - 1.18869I$	$3.25293 + 6.36321I$	$-12.47350 - 2.40837I$
$b = 1.72548 - 2.10674I$		
$u = -2.30418$		
$a = -0.741615$	$-18.6166$	$-11.2680$
$b = -0.517168$		

II.

$$I_2^u = \langle u^4 + 2u^3 - u^2 + b - u, \ u^4 + 3u^3 + u^2 + a - 2u - 1, \ u^5 + 3u^4 - 3u^2 + u - 1 \rangle$$

(i) **Arc colorings**

$$a_8 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_9 = \begin{pmatrix} 1 \\ u^2 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -u \\ -u^3 + u \end{pmatrix}$$

$$a_3 = \begin{pmatrix} -u^4 - 3u^3 - u^2 + 2u + 1 \\ -u^4 - 2u^3 + u^2 + u \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} u \\ u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} -2u^3 - 3u^2 + 3u \\ -u^3 - u^2 + 2u - 1 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -u^3 - 2u^2 + u + 1 \\ -u^3 - u^2 + 2u - 1 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} u^4 + 2u^3 - 2u^2 - 2u + 2 \\ -u^3 - 2u^2 + 2u \end{pmatrix}$$

$$a_2 = \begin{pmatrix} -2u^2 - 3u + 3 \\ -u^4 - 3u^3 + 2u + 1 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -u^4 - 3u^3 + 4u - 1 \\ u^4 + u^3 - u^2 + u - 2 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} -u^4 - 3u^3 + 4u \\ u^2 + u - 1 \end{pmatrix}$$

(ii) **Obstruction class = 1**

(iii) **Cusp Shapes** =  $-u^4 - 3u^3 - 4u^2 - u - 8$

**(iv) u-Polynomials at the component**

Crossings	u-Polynomials at each crossing
$c_1$	$u^5 - 7u^4 + 15u^3 - 20u^2 + 9u - 1$
$c_2$	$u^5 - u^4 - 3u^3 + 3u - 1$
$c_3$	$u^5 - u^4 - 3u^3 - 2u^2 - 3u - 1$
$c_4$	$u^5 - 2u^4 - u^3 + 4u^2 - 6u + 3$
$c_5, c_9$	$u^5 - 3u^4 + u^3 + u^2 - 4u + 1$
$c_6$	$u^5 + u^4 - 3u^3 + 3u + 1$
$c_7$	$u^5 + 2u^4 - u^3 - 4u^2 - 6u - 3$
$c_8$	$u^5 + 3u^4 - 3u^2 + u - 1$
$c_{10}$	$u^5 + 3u^4 + u^3 - u^2 - 4u - 1$
$c_{11}, c_{12}$	$u^5 - 3u^4 + 3u^2 + u + 1$

**(v) Riley Polynomials at the component**

Crossings	Riley Polynomials at each crossing
$c_1$	$y^5 - 19y^4 - 37y^3 - 144y^2 + 41y - 1$
$c_2, c_6$	$y^5 - 7y^4 + 15y^3 - 20y^2 + 9y - 1$
$c_3$	$y^5 - 7y^4 - y^3 + 12y^2 + 5y - 1$
$c_4, c_7$	$y^5 - 6y^4 + 5y^3 + 8y^2 + 12y - 9$
$c_5, c_9, c_{10}$	$y^5 - 7y^4 - y^3 - 3y^2 + 14y - 1$
$c_8, c_{11}, c_{12}$	$y^5 - 9y^4 + 20y^3 - 3y^2 - 5y - 1$

**(vi) Complex Volumes and Cusp Shapes**

Solutions to $I_2^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.896190$		
$a = -0.815182$	-2.59633	-14.9130
$b = -0.385277$		
$u = 0.116133 + 0.503198I$		
$a = 1.68814 + 1.26672I$	$2.78994 + 2.01434I$	$-6.94110 - 0.59350I$
$b = 0.005908 + 0.890217I$		
$u = 0.116133 - 0.503198I$		
$a = 1.68814 - 1.26672I$	$2.78994 - 2.01434I$	$-6.94110 + 0.59350I$
$b = 0.005908 - 0.890217I$		
$u = -1.78655$		
$a = 1.15452$	-12.8780	-12.0610
$b = 2.62235$		
$u = -2.34191$		
$a = -0.715612$	-19.7144	-19.1430
$b = -1.24889$		

### III. u-Polynomials

Crossings	u-Polynomials at each crossing
$c_1$	$(u^5 - 7u^4 + 15u^3 - 20u^2 + 9u - 1)$ $\cdot (u^8 + 7u^7 + 35u^6 + 244u^5 + 493u^4 + 885u^3 + 772u^2 + 608u + 64)$
$c_2$	$(u^5 - u^4 - 3u^3 + 3u - 1)$ $\cdot (u^8 - 9u^7 + 37u^6 - 88u^5 + 125u^4 - 101u^3 + 34u^2 + 8u - 8)$
$c_3$	$(u^5 - u^4 - 3u^3 - 2u^2 - 3u - 1)$ $\cdot (u^8 + 2u^7 + 4u^6 - 6u^5 + 10u^4 - 3u^3 + 5u^2 - u - 1)$
$c_4$	$(u^5 - 2u^4 - u^3 + 4u^2 - 6u + 3)$ $\cdot (u^8 - 3u^7 + 9u^6 - 2u^5 - 14u^3 - 19u^2 - 8u - 1)$
$c_5, c_9$	$(u^5 - 3u^4 + u^3 + u^2 - 4u + 1)$ $\cdot (u^8 + 2u^7 + 10u^6 + 51u^5 + 8u^4 + 28u^3 + 10u^2 + 2u + 1)$
$c_6$	$(u^5 + u^4 - 3u^3 + 3u + 1)$ $\cdot (u^8 - 9u^7 + 37u^6 - 88u^5 + 125u^4 - 101u^3 + 34u^2 + 8u - 8)$
$c_7$	$(u^5 + 2u^4 - u^3 - 4u^2 - 6u - 3)$ $\cdot (u^8 - 3u^7 + 9u^6 - 2u^5 - 14u^3 - 19u^2 - 8u - 1)$
$c_8$	$(u^5 + 3u^4 - 3u^2 + u - 1)$ $\cdot (u^8 + 4u^7 - u^6 - 22u^5 - 25u^4 + 4u^3 + 12u^2 - u - 1)$
$c_{10}$	$(u^5 + 3u^4 + u^3 - u^2 - 4u - 1)$ $\cdot (u^8 + 2u^7 + 10u^6 + 51u^5 + 8u^4 + 28u^3 + 10u^2 + 2u + 1)$
$c_{11}, c_{12}$	$(u^5 - 3u^4 + 3u^2 + u + 1)$ $\cdot (u^8 + 4u^7 - u^6 - 22u^5 - 25u^4 + 4u^3 + 12u^2 - u - 1)$

#### IV. Riley Polynomials

Crossings	Riley Polynomials at each crossing
$c_1$	$(y^5 - 19y^4 - 37y^3 - 144y^2 + 41y - 1)$ $\cdot (y^8 + 21y^7 + \dots - 270848y + 4096)$
$c_2, c_6$	$(y^5 - 7y^4 + 15y^3 - 20y^2 + 9y - 1)$ $\cdot (y^8 - 7y^7 + 35y^6 - 244y^5 + 493y^4 - 885y^3 + 772y^2 - 608y + 64)$
$c_3$	$(y^5 - 7y^4 - y^3 + 12y^2 + 5y - 1)$ $\cdot (y^8 + 4y^7 + 60y^6 + 66y^5 + 106y^4 + 71y^3 - y^2 - 11y + 1)$
$c_4, c_7$	$(y^5 - 6y^4 + 5y^3 + 8y^2 + 12y - 9)$ $\cdot (y^8 + 9y^7 + 69y^6 - 126y^5 - 448y^4 - 246y^3 + 137y^2 - 26y + 1)$
$c_5, c_9, c_{10}$	$(y^5 - 7y^4 - y^3 - 3y^2 + 14y - 1)$ $\cdot (y^8 + 16y^7 - 88y^6 - 2533y^5 - 2598y^4 - 808y^3 + 4y^2 + 16y + 1)$
$c_8, c_{11}, c_{12}$	$(y^5 - 9y^4 + 20y^3 - 3y^2 - 5y - 1)$ $\cdot (y^8 - 18y^7 + 127y^6 - 442y^5 + 783y^4 - 658y^3 + 202y^2 - 25y + 1)$