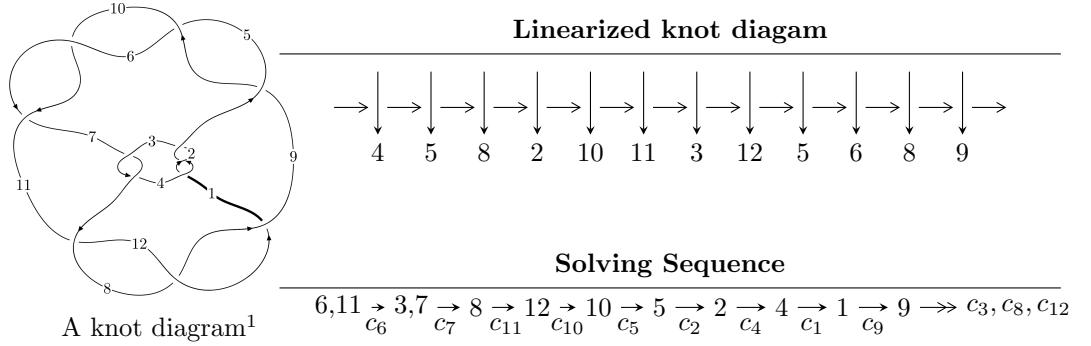


## $12n_{0688}$ ( $K12n_{0688}$ )



### Ideals for irreducible components<sup>2</sup> of $X_{\text{par}}$

$$I_1^u = \langle -1.45733 \times 10^{20}u^{26} - 2.14063 \times 10^{20}u^{25} + \dots + 3.64969 \times 10^{20}b + 1.19460 \times 10^{21}, \\ - 4.51116 \times 10^{20}u^{26} - 7.95319 \times 10^{20}u^{25} + \dots + 3.64969 \times 10^{20}a + 5.99946 \times 10^{21}, \\ u^{27} + 2u^{26} + \dots - 12u - 4 \rangle$$

$$I_2^u = \langle b - u + 1, -u^2 + a + 3, u^3 - u^2 - 2u + 1 \rangle$$

$$I_3^u = \langle -au + b + 1, 2a^2 + au + 2a - 2u - 3, u^2 - 2 \rangle$$

$$I_1^v = \langle a, b - v - 2, v^2 + 3v + 1 \rangle$$

\* 4 irreducible components of  $\dim_{\mathbb{C}} = 0$ , with total 36 representations.

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<sup>1</sup>The image of knot diagram is generated by the software “**Draw programme**” developed by Andrew Bartholomew(<http://www.layer8.co.uk/math/draw/index.htm#Running-draw>), where we modified some parts for our purpose(<https://github.com/CATsTAILs/LinksPainter>).

<sup>2</sup>All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

$$\text{I. } I_1^u = \langle -1.46 \times 10^{20}u^{26} - 2.14 \times 10^{20}u^{25} + \dots + 3.65 \times 10^{20}b + 1.19 \times 10^{21}, -4.51 \times 10^{20}u^{26} - 7.95 \times 10^{20}u^{25} + \dots + 3.65 \times 10^{20}a + 6.00 \times 10^{21}, u^{27} + 2u^{26} + \dots - 12u - 4 \rangle$$

(i) Arc colorings

$$\begin{aligned} a_6 &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ a_{11} &= \begin{pmatrix} 0 \\ u \end{pmatrix} \\ a_3 &= \begin{pmatrix} 1.23604u^{26} + 2.17914u^{25} + \dots + 8.25237u - 16.4383 \\ 0.399304u^{26} + 0.586525u^{25} + \dots - 3.39246u - 3.27317 \end{pmatrix} \\ a_7 &= \begin{pmatrix} 1 \\ u^2 \end{pmatrix} \\ a_8 &= \begin{pmatrix} 0.738918u^{26} + 1.22294u^{25} + \dots + 2.14974u - 8.57454 \\ 0.353041u^{26} + 0.631631u^{25} + \dots - 2.16585u - 3.44416 \end{pmatrix} \\ a_{12} &= \begin{pmatrix} 0.175121u^{26} + 0.148966u^{25} + \dots - 7.10327u + 0.964438 \\ 0.560998u^{26} + 0.740275u^{25} + \dots - 2.78768u - 4.16594 \end{pmatrix} \\ a_{10} &= \begin{pmatrix} u \\ u \end{pmatrix} \\ a_5 &= \begin{pmatrix} -u^2 + 1 \\ -u^2 \end{pmatrix} \\ a_2 &= \begin{pmatrix} 1.66235u^{26} + 2.73772u^{25} + \dots + 6.10273u - 20.0632 \\ 0.563386u^{26} + 0.613561u^{25} + \dots - 4.52635u - 3.30149 \end{pmatrix} \\ a_4 &= \begin{pmatrix} 0.670706u^{26} + 1.16883u^{25} + \dots + 9.64782u - 9.46907 \\ -0.635285u^{26} - 0.677229u^{25} + \dots + 4.25702u + 3.47590 \end{pmatrix} \\ a_1 &= \begin{pmatrix} -0.0973141u^{26} + 0.0651568u^{25} + \dots + 7.52379u - 2.64979 \\ -0.591878u^{26} - 0.655073u^{25} + \dots + 3.31129u + 3.50017 \end{pmatrix} \\ a_9 &= \begin{pmatrix} -u^3 + 2u \\ -u^3 + u \end{pmatrix} \end{aligned}$$

(ii) Obstruction class = -1

$$\text{(iii) Cusp Shapes} = \frac{237495891437804361646}{91242178035245717881}u^{26} + \frac{405928781365325849828}{91242178035245717881}u^{25} + \dots + \frac{4084274379373058773658}{91242178035245717881}u - \frac{5388560215252919173048}{91242178035245717881}$$

**(iv) u-Polynomials at the component**

Crossings	u-Polynomials at each crossing
$c_1, c_2, c_4$	$u^{27} - 7u^{26} + \cdots - 9u - 1$
$c_3, c_7$	$u^{27} - 2u^{26} + \cdots - 52u + 8$
$c_5, c_6, c_9$ $c_{10}$	$u^{27} - 2u^{26} + \cdots - 12u + 4$
$c_8, c_{11}, c_{12}$	$u^{27} + 4u^{26} + \cdots - 57u - 9$

**(v) Riley Polynomials at the component**

Crossings	Riley Polynomials at each crossing
$c_1, c_2, c_4$	$y^{27} - 15y^{26} + \cdots + 103y - 1$
$c_3, c_7$	$y^{27} + 12y^{26} + \cdots + 3920y - 64$
$c_5, c_6, c_9$ $c_{10}$	$y^{27} - 24y^{26} + \cdots + 432y - 16$
$c_8, c_{11}, c_{12}$	$y^{27} - 10y^{26} + \cdots + 1179y - 81$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.103210 + 1.061510I$		
$a = 0.243591 + 1.283730I$	$5.33632 - 0.36665I$	$-12.32525 - 0.05039I$
$b = 0.314942 + 0.159811I$		
$u = -0.103210 - 1.061510I$		
$a = 0.243591 - 1.283730I$	$5.33632 + 0.36665I$	$-12.32525 + 0.05039I$
$b = 0.314942 - 0.159811I$		
$u = -0.903239 + 0.167691I$		
$a = -1.45135 - 0.68497I$	$-3.12027 + 0.78467I$	$-18.4367 - 3.3729I$
$b = -0.485625 - 0.865359I$		
$u = -0.903239 - 0.167691I$		
$a = -1.45135 + 0.68497I$	$-3.12027 - 0.78467I$	$-18.4367 + 3.3729I$
$b = -0.485625 + 0.865359I$		
$u = 0.337221 + 1.048840I$		
$a = 0.052763 - 1.351640I$	$3.59374 - 7.31725I$	$-14.7365 + 5.0472I$
$b = 0.215415 - 0.194118I$		
$u = 0.337221 - 1.048840I$		
$a = 0.052763 + 1.351640I$	$3.59374 + 7.31725I$	$-14.7365 - 5.0472I$
$b = 0.215415 + 0.194118I$		
$u = 1.104930 + 0.368271I$		
$a = -0.268980 + 0.135914I$	$-3.05220 - 3.96537I$	$-17.7910 + 4.2991I$
$b = 0.714285 - 1.043610I$		
$u = 1.104930 - 0.368271I$		
$a = -0.268980 - 0.135914I$	$-3.05220 + 3.96537I$	$-17.7910 - 4.2991I$
$b = 0.714285 + 1.043610I$		
$u = 1.020210 + 0.720557I$		
$a = 0.780740 - 0.476181I$	$1.52420 + 1.21028I$	$-14.6857 - 1.0471I$
$b = 1.045480 - 0.589623I$		
$u = 1.020210 - 0.720557I$		
$a = 0.780740 + 0.476181I$	$1.52420 - 1.21028I$	$-14.6857 + 1.0471I$
$b = 1.045480 + 0.589623I$		

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -1.33719$		
$a = 0.790731$	-14.1221	-16.5250
$b = -0.340332$		
$u = -1.374500 + 0.240422I$		
$a = -0.496066 - 0.228689I$	$-6.49373 + 0.53770I$	$-14.7868 - 0.8041I$
$b = -1.169080 - 0.741326I$		
$u = -1.374500 - 0.240422I$		
$a = -0.496066 + 0.228689I$	$-6.49373 - 0.53770I$	$-14.7868 + 0.8041I$
$b = -1.169080 + 0.741326I$		
$u = 1.40105$		
$a = -11.6087$	-8.19904	-208.640
$b = -22.6375$		
$u = -1.295330 + 0.551421I$		
$a = 0.862994 + 0.773421I$	$1.64132 + 6.06050I$	$-15.3648 - 4.1353I$
$b = 1.25669 + 1.17478I$		
$u = -1.295330 - 0.551421I$		
$a = 0.862994 - 0.773421I$	$1.64132 - 6.06050I$	$-15.3648 + 4.1353I$
$b = 1.25669 - 1.17478I$		
$u = 0.279491 + 0.475963I$		
$a = 0.278696 - 0.748546I$	$-0.648673 + 0.468512I$	$-12.96489 + 0.08688I$
$b = -0.794845 + 0.094176I$		
$u = 0.279491 - 0.475963I$		
$a = 0.278696 + 0.748546I$	$-0.648673 - 0.468512I$	$-12.96489 - 0.08688I$
$b = -0.794845 - 0.094176I$		
$u = -1.45969$		
$a = -1.05208$	-6.73578	-12.1710
$b = -1.81866$		
$u = 1.44790 + 0.52184I$		
$a = -0.710802 + 0.417384I$	$0.46040 - 5.31882I$	$-15.6377 + 3.3723I$
$b = -1.42465 + 1.21712I$		

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 1.44790 - 0.52184I$		
$a = -0.710802 - 0.417384I$	$0.46040 + 5.31882I$	$-15.6377 - 3.3723I$
$b = -1.42465 - 1.21712I$		
$u = -0.459185$		
$a = 1.01896$	-10.8656	-30.2970
$b = 1.69587$		
$u = -1.51780 + 0.43184I$		
$a = -0.849204 - 0.755740I$	$-2.32586 + 12.67780I$	$-18.7053 - 6.5054I$
$b = -1.65455 - 1.79207I$		
$u = -1.51780 - 0.43184I$		
$a = -0.849204 + 0.755740I$	$-2.32586 - 12.67780I$	$-18.7053 + 6.5054I$
$b = -1.65455 + 1.79207I$		
$u = 0.379857$		
$a = 0.653894$	-0.575852	-17.0320
$b = -0.255787$		
$u = -0.278308$		
$a = -8.72003$	-2.85525	-50.3790
$b = -0.183695$		
$u = 1.76214$		
$a = 0.0324794$	-19.5641	-33.0880
$b = -0.496020$		

$$\text{II. } I_2^u = \langle b - u + 1, -u^2 + a + 3, u^3 - u^2 - 2u + 1 \rangle$$

(i) Arc colorings

$$a_6 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_3 = \begin{pmatrix} u^2 - 3 \\ u - 1 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} 1 \\ u^2 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} 1 \\ u^2 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} -u \\ -u^2 - u + 1 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} u \\ u \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -u^2 + 1 \\ -u^2 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} 2u^2 - 4 \\ u^2 + u - 1 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} u^2 - 3 \\ u - 1 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} u^2 - 1 \\ u^2 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -u^2 + 1 \\ -u^2 - u + 1 \end{pmatrix}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes =  $u^2 + 4u - 16$

**(iv) u-Polynomials at the component**

Crossings	u-Polynomials at each crossing
$c_1, c_2$	$(u - 1)^3$
$c_3, c_7$	$u^3$
$c_4$	$(u + 1)^3$
$c_5, c_6, c_8$	$u^3 - u^2 - 2u + 1$
$c_9, c_{10}, c_{11}$ $c_{12}$	$u^3 + u^2 - 2u - 1$

**(v) Riley Polynomials at the component**

Crossings	Riley Polynomials at each crossing
$c_1, c_2, c_4$	$(y - 1)^3$
$c_3, c_7$	$y^3$
$c_5, c_6, c_8$ $c_9, c_{10}, c_{11}$ $c_{12}$	$y^3 - 5y^2 + 6y - 1$

**(vi) Complex Volumes and Cusp Shapes**

Solutions to $I_2^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -1.24698$		
$a = -1.44504$	-7.98968	-19.4330
$b = -2.24698$		
$u = 0.445042$		
$a = -2.80194$	-2.34991	-14.0220
$b = -0.554958$		
$u = 1.80194$		
$a = 0.246980$	-19.2692	-5.54530
$b = 0.801938$		

$$\text{III. } I_3^u = \langle -au + b + 1, 2a^2 + au + 2a - 2u - 3, u^2 - 2 \rangle$$

(i) Arc colorings

$$a_6 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_3 = \begin{pmatrix} a \\ au - 1 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -\frac{1}{2}u \\ -au + 2a - u + 2 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} -\frac{1}{2}u \\ -au + 2a + 2 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} u \\ u \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -1 \\ -2 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} au - a - 1 \\ 3au - 4a - 3 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} -au + 2a - \frac{1}{2}u \\ -3au + 6a - u + 2 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -\frac{1}{2}u \\ -au + 2a - u + 2 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} 0 \\ -u \end{pmatrix}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes = -24

**(iv) u-Polynomials at the component**

Crossings	u-Polynomials at each crossing
$c_1, c_2, c_7$	$(u^2 + u - 1)^2$
$c_3, c_4$	$(u^2 - u - 1)^2$
$c_5, c_6, c_9$ $c_{10}$	$(u^2 - 2)^2$
$c_8$	$(u + 1)^4$
$c_{11}, c_{12}$	$(u - 1)^4$

**(v) Riley Polynomials at the component**

Crossings	Riley Polynomials at each crossing
$c_1, c_2, c_3$ $c_4, c_7$	$(y^2 - 3y + 1)^2$
$c_5, c_6, c_9$ $c_{10}$	$(y - 2)^4$
$c_8, c_{11}, c_{12}$	$(y - 1)^4$

**(vi) Complex Volumes and Cusp Shapes**

Solutions to $I_3^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 1.41421$		
$a = 1.05505$	-15.4624	-24.0000
$b = 0.492066$		
$u = -1.41421$		
$a = -2.76216$	-7.56670	-24.0000
$b = -4.90628$		
$u = -1.41421$		
$a = -0.473911$	-7.56670	-24.0000
$b = -0.329788$		
$u = -1.41421$		
$a = 0.181018$	-15.4624	-24.0000
$b = -1.25600$		

$$\text{IV. } I_1^v = \langle a, b - v - 2, v^2 + 3v + 1 \rangle$$

(i) Arc colorings

$$a_6 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} v \\ 0 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} 0 \\ v+2 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} 1 \\ v+3 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} v-1 \\ -v-3 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} v \\ 0 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} v+2 \\ v+2 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} -v-2 \\ -v-3 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -1 \\ -v-3 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} v \\ 0 \end{pmatrix}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes = -6

**(iv) u-Polynomials at the component**

Crossings	u-Polynomials at each crossing
$c_1, c_2, c_3$	$u^2 + u - 1$
$c_4, c_7$	$u^2 - u - 1$
$c_5, c_6, c_9$ $c_{10}$	$u^2$
$c_8$	$(u - 1)^2$
$c_{11}, c_{12}$	$(u + 1)^2$

**(v) Riley Polynomials at the component**

Crossings	Riley Polynomials at each crossing
$c_1, c_2, c_3$ $c_4, c_7$	$y^2 - 3y + 1$
$c_5, c_6, c_9$ $c_{10}$	$y^2$
$c_8, c_{11}, c_{12}$	$(y - 1)^2$

**(vi) Complex Volumes and Cusp Shapes**

Solutions to $I_1^v$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$v = -0.381966$		
$a = 0$	-10.5276	-6.00000
$b = 1.61803$		
$v = -2.61803$		
$a = 0$	-2.63189	-6.00000
$b = -0.618034$		

## V. u-Polynomials

Crossings	u-Polynomials at each crossing
$c_1, c_2$	$((u - 1)^3)(u^2 + u - 1)^3(u^{27} - 7u^{26} + \dots - 9u - 1)$
$c_3$	$u^3(u^2 - u - 1)^2(u^2 + u - 1)(u^{27} - 2u^{26} + \dots - 52u + 8)$
$c_4$	$((u + 1)^3)(u^2 - u - 1)^3(u^{27} - 7u^{26} + \dots - 9u - 1)$
$c_5, c_6$	$u^2(u^2 - 2)^2(u^3 - u^2 - 2u + 1)(u^{27} - 2u^{26} + \dots - 12u + 4)$
$c_7$	$u^3(u^2 - u - 1)(u^2 + u - 1)^2(u^{27} - 2u^{26} + \dots - 52u + 8)$
$c_8$	$((u - 1)^2)(u + 1)^4(u^3 - u^2 - 2u + 1)(u^{27} + 4u^{26} + \dots - 57u - 9)$
$c_9, c_{10}$	$u^2(u^2 - 2)^2(u^3 + u^2 - 2u - 1)(u^{27} - 2u^{26} + \dots - 12u + 4)$
$c_{11}, c_{12}$	$((u - 1)^4)(u + 1)^2(u^3 + u^2 - 2u - 1)(u^{27} + 4u^{26} + \dots - 57u - 9)$

## VI. Riley Polynomials

Crossings	Riley Polynomials at each crossing
$c_1, c_2, c_4$	$((y - 1)^3)(y^2 - 3y + 1)^3(y^{27} - 15y^{26} + \cdots + 103y - 1)$
$c_3, c_7$	$y^3(y^2 - 3y + 1)^3(y^{27} + 12y^{26} + \cdots + 3920y - 64)$
$c_5, c_6, c_9$ $c_{10}$	$y^2(y - 2)^4(y^3 - 5y^2 + 6y - 1)(y^{27} - 24y^{26} + \cdots + 432y - 16)$
$c_8, c_{11}, c_{12}$	$((y - 1)^6)(y^3 - 5y^2 + 6y - 1)(y^{27} - 10y^{26} + \cdots + 1179y - 81)$