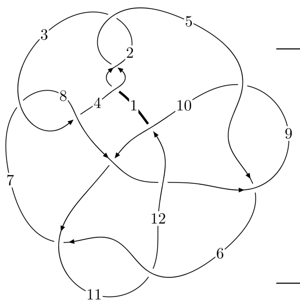
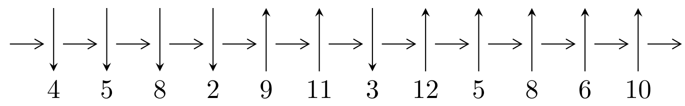
# $12n_{0690}$ $(K12n_{0690})$




A knot diagram[1]

**Linearized knot diagam**




**Solving Sequence**

$$2,5 \underset{c_4}{\longrightarrow} 4 \underset{c_1}{\rightarrow} 1,10 \underset{c_9}{\rightarrow} 9 \underset{c_5}{\longrightarrow} 6 \underset{c_{12}}{\rightarrow} 12 \underset{c_8}{\rightarrow} 8 \underset{c_3}{\longrightarrow} 3 \underset{c_7}{\longrightarrow} 7 \underset{c_{11}}{\rightarrow} 11 \twoheadrightarrow c_2, c_6, c_{10}$$

**Ideals for irreducible components[2] of $X_{\mathbf{par}}$**

$$\begin{aligned}
I_1^u = \langle & -2.93346 \times 10^{17} u^{21} + 1.10518 \times 10^{18} u^{20} + \cdots + 6.77435 \times 10^{18} b + 2.05511 \times 10^{18}, \\
& -4.34516 \times 10^{16} u^{21} + 5.25657 \times 10^{17} u^{20} + \cdots + 6.77435 \times 10^{18} a - 3.58257 \times 10^{19}, \\
& u^{22} - 5u^{21} + \cdots + 108u + 16 \rangle \\
I_2^u = \langle & -u^4 a^2 + 5u^4 a + \cdots + 3a - 3,\ u^4 a^3 + 5u^4 a^2 + \cdots + 27a + 31,\ u^5 - 2u^4 + 2u^3 + u^2 - u + 1 \rangle \\
I_3^u = \langle & 2a^2 + 2b + 3a + 2,\ 4a^3 + 4a^2 + 5a + 4,\ u + 1 \rangle \\
I_4^u = \langle & u^{12} - 3u^{11} + u^{10} + 6u^9 - 6u^8 - 5u^7 + 7u^6 + 2u^5 - 2u^4 + 2u^3 - u^2 + b - 3u - 1, \\
& -u^{12} + 4u^{11} - 5u^{10} - 2u^9 + 12u^8 - 12u^7 + 3u^6 + 6u^5 - 12u^4 + 7u^3 + a + 1, \\
& u^{13} - 5u^{12} + 8u^{11} + u^{10} - 18u^9 + 18u^8 + 2u^7 - 13u^6 + 10u^5 - 5u^4 - 3u^3 + 3u^2 + u + 1 \rangle \\
I_5^u = \langle & b^2 + ba - a,\ a^2 + a + 1,\ u + 1 \rangle
\end{aligned}$$

* 5 irreducible components of $\dim_{\mathbb{C}} = 0$, with total 62 representations.

[1] The image of knot diagram is generated by the software "**Draw programme**" developed by Andrew Bartholomew(`http://www.layer8.co.uk/maths/draw/index.htm#Running-draw`), where we modified some parts for our purpose(`https://github.com/CATsTAILs/LinksPainter`).

[2] All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

**I.** $I_1^u = \langle -2.93\times10^{17}u^{21} + 1.11\times10^{18}u^{20} + \cdots + 6.77\times10^{18}b + 2.06\times10^{18},\ -4.35\times10^{16}u^{21} + 5.26\times10^{17}u^{20} + \cdots + 6.77\times10^{18}a - 3.58\times10^{19},\ u^{22} - 5u^{21} + \cdots + 108u + 16\rangle$

**(i) Arc colorings**

$$a_2 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$
$$a_5 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$
$$a_4 = \begin{pmatrix} 1 \\ -u^2 \end{pmatrix}$$
$$a_1 = \begin{pmatrix} u \\ -u^3 + u \end{pmatrix}$$
$$a_{10} = \begin{pmatrix} 0.00641415u^{21} - 0.0775953u^{20} + \cdots + 4.28301u + 5.28844 \\ 0.0433025u^{21} - 0.163141u^{20} + \cdots - 5.64620u - 0.303366 \end{pmatrix}$$
$$a_9 = \begin{pmatrix} -0.0368883u^{21} + 0.0855460u^{20} + \cdots + 9.92921u + 5.59180 \\ 0.0433025u^{21} - 0.163141u^{20} + \cdots - 5.64620u - 0.303366 \end{pmatrix}$$
$$a_6 = \begin{pmatrix} -0.0252335u^{21} + 0.102422u^{20} + \cdots + 0.204852u - 0.803579 \\ 0.0147201u^{21} - 0.0585005u^{20} + \cdots - 0.406958u - 0.266094 \end{pmatrix}$$
$$a_{12} = \begin{pmatrix} 0.0213809u^{21} - 0.116096u^{20} + \cdots + 1.68488u + 3.12427 \\ -0.0129002u^{21} + 0.0465450u^{20} + \cdots + 0.617134u + 0.230969 \end{pmatrix}$$
$$a_8 = \begin{pmatrix} 0.0620474u^{21} - 0.266688u^{20} + \cdots - 4.65907u + 1.63431 \\ -0.0435486u^{21} + 0.186762u^{20} + \cdots + 5.06681u + 0.992758 \end{pmatrix}$$
$$a_3 = \begin{pmatrix} -u \\ u \end{pmatrix}$$
$$a_7 = \begin{pmatrix} 0.0771001u^{21} - 0.323949u^{20} + \cdots - 6.31235u + 1.43322 \\ -0.0586013u^{21} + 0.244022u^{20} + \cdots + 6.72009u + 1.19384 \end{pmatrix}$$
$$a_{11} = \begin{pmatrix} 0.122920u^{21} - 0.520465u^{20} + \cdots - 8.71637u + 2.14038 \\ -0.0963555u^{21} + 0.373312u^{20} + \cdots + 10.0845u + 1.56072 \end{pmatrix}$$

**(ii) Obstruction class** $= -1$

**(iii) Cusp Shapes** $= \frac{143551482205654407 69}{27097382097032209472}u^{21} - \frac{32455260689474630771}{13548691048516104736}u^{20} + \cdots - \frac{22427580659478826 2169}{6774345524258052368}u + \frac{14951472397892805561}{1693586381064513092}$

**(iv) u-Polynomials at the component**

| Crossings | u-Polynomials at each crossing |
|---|---|
| $c_1, c_2, c_4$ | $u^{22} - 5u^{21} + \cdots + 108u + 16$ |
| $c_3, c_7$ | $u^{22} - 6u^{21} + \cdots + 160u - 128$ |
| $c_5, c_6, c_9$ $c_{11}$ | $u^{22} + 6u^{20} + \cdots - u - 1$ |
| $c_8$ | $u^{22} - 14u^{21} + \cdots - 464u + 32$ |
| $c_{10}, c_{12}$ | $u^{22} + 3u^{21} + \cdots + u + 1$ |

**(v) Riley Polynomials at the component**

| Crossings | Riley Polynomials at each crossing |
|---|---|
| $c_1, c_2, c_4$ | $y^{22} - 11y^{21} + \cdots - 10352y + 256$ |
| $c_3, c_7$ | $y^{22} + 12y^{21} + \cdots - 289792y + 16384$ |
| $c_5, c_6, c_9$<br>$c_{11}$ | $y^{22} + 12y^{21} + \cdots - 15y + 1$ |
| $c_8$ | $y^{22} + 6y^{21} + \cdots - 36608y + 1024$ |
| $c_{10}, c_{12}$ | $y^{22} - 37y^{21} + \cdots - 141y + 1$ |

**(vi) Complex Volumes and Cusp Shapes**

| Solutions to $I_1^u$ | $\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$ | Cusp shape |
|---|---|---|
| $u = -0.593822 + 0.658603I$<br>$a = -0.232154 + 0.275565I$<br>$b = -0.443126 + 0.548064I$ | $-1.76571 + 0.41277I$ | $-1.52576 - 0.47761I$ |
| $u = -0.593822 - 0.658603I$<br>$a = -0.232154 - 0.275565I$<br>$b = -0.443126 - 0.548064I$ | $-1.76571 - 0.41277I$ | $-1.52576 + 0.47761I$ |
| $u = -0.871168$<br>$a = -0.718681$<br>$b = -0.283731$ | $-1.22854$ | $-10.9960$ |
| $u = 1.143630 + 0.105102I$<br>$a = -0.057070 - 0.907033I$<br>$b = -0.19308 + 1.40671I$ | $-11.40350 - 5.37019I$ | $-14.2310 + 9.8207I$ |
| $u = 1.143630 - 0.105102I$<br>$a = -0.057070 + 0.907033I$<br>$b = -0.19308 - 1.40671I$ | $-11.40350 + 5.37019I$ | $-14.2310 - 9.8207I$ |
| $u = -0.551985 + 0.412677I$<br>$a = 2.14943 - 0.33347I$<br>$b = 0.514816 + 0.271439I$ | $0.924854 + 0.158726I$ | $4.34440 - 6.69903I$ |
| $u = -0.551985 - 0.412677I$<br>$a = 2.14943 + 0.33347I$<br>$b = 0.514816 - 0.271439I$ | $0.924854 - 0.158726I$ | $4.34440 + 6.69903I$ |
| $u = -0.864951 + 1.007370I$<br>$a = -1.007060 + 0.230412I$<br>$b = -0.863800 - 0.926848I$ | $-1.67557 + 5.53623I$ | $-0.55960 - 5.41231I$ |
| $u = -0.864951 - 1.007370I$<br>$a = -1.007060 - 0.230412I$<br>$b = -0.863800 + 0.926848I$ | $-1.67557 - 5.53623I$ | $-0.55960 + 5.41231I$ |
| $u = 0.636703 + 1.182420I$<br>$a = -0.620154 - 0.137775I$<br>$b = -0.861446 - 0.939694I$ | $7.30571 - 0.11649I$ | $1.357790 - 0.220682I$ |

| Solutions to $I_1^u$ | $\sqrt{-1}(\mathrm{vol}+\sqrt{-1}CS)$ | Cusp shape |
|---|---|---|
| $u = 0.636703 - 1.182420I$<br>$a = -0.620154 + 0.137775I$<br>$b = -0.861446 + 0.939694I$ | $7.30571 + 0.11649I$ | $1.357790 + 0.220682I$ |
| $u = 1.46880 + 0.23288I$<br>$a = 0.344293 + 0.498724I$<br>$b = -0.233614 - 0.763468I$ | $-8.21433 - 3.59803I$ | $-0.94605 + 7.25200I$ |
| $u = 1.46880 - 0.23288I$<br>$a = 0.344293 - 0.498724I$<br>$b = -0.233614 + 0.763468I$ | $-8.21433 + 3.59803I$ | $-0.94605 - 7.25200I$ |
| $u = 1.29164 + 0.77827I$<br>$a = -1.062860 - 0.754268I$<br>$b = -0.540251 + 1.195250I$ | $5.05931 - 7.06255I$ | $-1.67212 + 4.37965I$ |
| $u = 1.29164 - 0.77827I$<br>$a = -1.062860 + 0.754268I$<br>$b = -0.540251 - 1.195250I$ | $5.05931 + 7.06255I$ | $-1.67212 - 4.37965I$ |
| $u = -1.51290 + 0.00522I$<br>$a = -0.178858 + 0.739842I$<br>$b = 0.667548 - 0.729025I$ | $-3.75642 - 2.26076I$ | $-0.68469 + 3.46770I$ |
| $u = -1.51290 - 0.00522I$<br>$a = -0.178858 - 0.739842I$<br>$b = 0.667548 + 0.729025I$ | $-3.75642 + 2.26076I$ | $-0.68469 - 3.46770I$ |
| $u = 0.65142 + 1.38092I$<br>$a = 0.480434 + 0.328699I$<br>$b = 1.05295 + 1.31649I$ | $5.55369 + 6.92717I$ | $0.41712 - 3.63279I$ |
| $u = 0.65142 - 1.38092I$<br>$a = 0.480434 - 0.328699I$<br>$b = 1.05295 - 1.31649I$ | $5.55369 - 6.92717I$ | $0.41712 + 3.63279I$ |
| $u = 1.35088 + 0.88382I$<br>$a = 1.167610 + 0.664320I$<br>$b = 0.82472 - 1.52688I$ | $3.1876 - 14.9988I$ | $-1.61073 + 7.00884I$ |

| Solutions to $I_1^u$ | $\sqrt{-1}(\mathrm{vol} + \sqrt{-1}CS)$ | Cusp shape |
|---|---|---|
| $u = 1.35088 - 0.88382I$<br>$a = 1.167610 - 0.664320I$<br>$b = 0.82472 + 1.52688I$ | $3.1876 + 14.9988I$ | $-1.61073 - 7.00884I$ |
| $u = -0.167658$<br>$a = 4.50144$<br>$b = 0.434292$ | $0.927721$ | $12.4670$ |

**II.** $\boldsymbol{I_2^u = \langle -u^4a^2 + 5u^4a + \cdots + 3a - 3,\ u^4a^3 + 5u^4a^2 + \cdots + 27a + 31,\ u^5 - 2u^4 + 2u^3 + u^2 - u + 1\rangle}$

**(i) Arc colorings**

$$a_2 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$
$$a_5 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$
$$a_4 = \begin{pmatrix} 1 \\ -u^2 \end{pmatrix}$$
$$a_1 = \begin{pmatrix} u \\ -u^3 + u \end{pmatrix}$$
$$a_{10} = \begin{pmatrix} a \\ \frac{1}{2}u^4a^2 - \frac{5}{2}u^4a + \cdots - \frac{3}{2}a + \frac{3}{2} \end{pmatrix}$$
$$a_9 = \begin{pmatrix} -\frac{1}{2}u^4a^2 + \frac{5}{2}u^4a + \cdots + \frac{5}{2}a - \frac{3}{2} \\ \frac{1}{2}u^4a^2 - \frac{5}{2}u^4a + \cdots - \frac{3}{2}a + \frac{3}{2} \end{pmatrix}$$
$$a_6 = \begin{pmatrix} 2u^4a^3 - 3u^4a^2 + \cdots - a^2 - 6 \\ -\frac{5}{2}u^4a^3 + \frac{7}{2}u^4a^2 + \cdots - \frac{9}{2}a + 5 \end{pmatrix}$$
$$a_{12} = \begin{pmatrix} -\frac{1}{2}u^4a^3 + \frac{1}{2}u^4a^2 + \cdots - \frac{7}{2}a - 7 \\ \frac{3}{2}u^4a^3 - \frac{5}{2}u^4a^2 + \cdots + \frac{5}{2}a - 5 \end{pmatrix}$$
$$a_8 = \begin{pmatrix} -u^4 + 2u^3 - u^2 - 2u + 1 \\ -u^3 + u^2 - 1 \end{pmatrix}$$
$$a_3 = \begin{pmatrix} -u \\ u \end{pmatrix}$$
$$a_7 = \begin{pmatrix} -u^4 + 3u^3 - u^2 - 2u + 2 \\ -2u^3 + u^2 - 2 \end{pmatrix}$$
$$a_{11} = \begin{pmatrix} -\frac{3}{2}u^4a^2 + \frac{3}{2}u^4a + \cdots + \frac{1}{2}a - \frac{21}{2} \\ u^4a^3 - u^4a^2 + \cdots - 4a^2 + 3 \end{pmatrix}$$

**(ii) Obstruction class** $= -1$

**(iii) Cusp Shapes** $= 4u^4a^2 - 4a^3u - 4a^2u^2 + 4u^3a - 16u^4 - 4a^3 + 12a^2u - 8u^2a + 26u^3 + 8a^2 + 16au - 28u^2 - 4a - 2u - 24$

**(iv) u-Polynomials at the component**

| Crossings | u-Polynomials at each crossing |
|---|---|
| $c_1, c_2, c_4$ | $(u^5 - 2u^4 + 2u^3 + u^2 - u + 1)^4$ |
| $c_3, c_7$ | $(u^5 + u^4 + 5u^3 + u^2 + 2u - 2)^4$ |
| $c_5, c_6, c_9$ $c_{11}$ | $u^{20} - 2u^{19} + \cdots + 150u + 103$ |
| $c_8$ | $(u^2 + u + 1)^{10}$ |
| $c_{10}, c_{12}$ | $u^{20} + 2u^{19} + \cdots + 13224u + 2521$ |

**(v) Riley Polynomials at the component**

| Crossings | Riley Polynomials at each crossing |
|---|---|
| $c_1, c_2, c_4$ | $(y^5 + 6y^3 - y^2 - y - 1)^4$ |
| $c_3, c_7$ | $(y^5 + 9y^4 + 27y^3 + 23y^2 + 8y - 4)^4$ |
| $c_5, c_6, c_9$ $c_{11}$ | $y^{20} + 6y^{19} + \cdots + 110988y + 10609$ |
| $c_8$ | $(y^2 + y + 1)^{10}$ |
| $c_{10}, c_{12}$ | $y^{20} - 18y^{19} + \cdots + 37696544y + 6355441$ |

**(vi) Complex Volumes and Cusp Shapes**

| Solutions to $I_2^u$ | $\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$ | Cusp shape |
|---|---|---|
| $u = -0.833800$<br>$a = -2.41812 + 0.15016I$<br>$b = -0.272476 - 1.364140I$ | $-6.13845 + 2.02988I$ | $-10.94304 - 3.46410I$ |
| $u = -0.833800$<br>$a = -2.41812 - 0.15016I$<br>$b = -0.272476 + 1.364140I$ | $-6.13845 - 2.02988I$ | $-10.94304 + 3.46410I$ |
| $u = -0.833800$<br>$a = -0.91730 + 5.62696I$<br>$b = 0.409925 + 1.126070I$ | $-6.13845 + 2.02988I$ | $-10.94304 - 3.46410I$ |
| $u = -0.833800$<br>$a = -0.91730 - 5.62696I$<br>$b = 0.409925 - 1.126070I$ | $-6.13845 - 2.02988I$ | $-10.94304 + 3.46410I$ |
| $u = 0.317129 + 0.618084I$<br>$a = -1.226670 - 0.088438I$<br>$b = -0.05655 + 1.55273I$ | $-3.08342 + 0.92097I$ | $0.36548 - 1.42298I$ |
| $u = 0.317129 + 0.618084I$<br>$a = -1.283840 - 0.195995I$<br>$b = -1.102440 + 0.773065I$ | $-3.08342 - 3.13880I$ | $0.36548 + 5.50523I$ |
| $u = 0.317129 + 0.618084I$<br>$a = 1.42128 - 0.76900I$<br>$b = -0.035904 - 0.509258I$ | $-3.08342 + 0.92097I$ | $0.36548 - 1.42298I$ |
| $u = 0.317129 + 0.618084I$<br>$a = 1.92910 + 0.79325I$<br>$b = 0.244995 - 1.374870I$ | $-3.08342 - 3.13880I$ | $0.36548 + 5.50523I$ |
| $u = 0.317129 - 0.618084I$<br>$a = -1.226670 + 0.088438I$<br>$b = -0.05655 - 1.55273I$ | $-3.08342 - 0.92097I$ | $0.36548 + 1.42298I$ |
| $u = 0.317129 - 0.618084I$<br>$a = -1.283840 + 0.195995I$<br>$b = -1.102440 - 0.773065I$ | $-3.08342 + 3.13880I$ | $0.36548 - 5.50523I$ |

| Solutions to $I_2^u$ | $\sqrt{-1}(\mathrm{vol}+\sqrt{-1}CS)$ | Cusp shape |
|---|---|---|
| $u = 0.317129 - 0.618084I$<br>$a = 1.42128 + 0.76900I$<br>$b = -0.035904 + 0.509258I$ | $-3.08342 - 0.92097I$ | $0.36548 + 1.42298I$ |
| $u = 0.317129 - 0.618084I$<br>$a = 1.92910 - 0.79325I$<br>$b = 0.244995 + 1.374870I$ | $-3.08342 + 3.13880I$ | $0.36548 - 5.50523I$ |
| $u = 1.09977 + 1.12945I$<br>$a = 0.969002 + 0.567215I$<br>$b = 1.49281 - 1.00208I$ | $6.97511 - 2.09502I$ | $-0.89396 - 1.30967I$ |
| $u = 1.09977 + 1.12945I$<br>$a = -1.097140 - 0.256413I$<br>$b = -0.722774 + 1.025520I$ | $6.97511 - 6.15479I$ | $-0.89396 + 5.61853I$ |
| $u = 1.09977 + 1.12945I$<br>$a = 0.555611 + 0.563935I$<br>$b = 1.77894 + 0.45753I$ | $6.97511 - 6.15479I$ | $-0.89396 + 5.61853I$ |
| $u = 1.09977 + 1.12945I$<br>$a = -0.431917 - 0.251999I$<br>$b = -0.736535 - 0.654115I$ | $6.97511 - 2.09502I$ | $-0.89396 - 1.30967I$ |
| $u = 1.09977 - 1.12945I$<br>$a = 0.969002 - 0.567215I$<br>$b = 1.49281 + 1.00208I$ | $6.97511 + 2.09502I$ | $-0.89396 + 1.30967I$ |
| $u = 1.09977 - 1.12945I$<br>$a = -1.097140 + 0.256413I$<br>$b = -0.722774 - 1.025520I$ | $6.97511 + 6.15479I$ | $-0.89396 - 5.61853I$ |
| $u = 1.09977 - 1.12945I$<br>$a = 0.555611 - 0.563935I$<br>$b = 1.77894 - 0.45753I$ | $6.97511 + 6.15479I$ | $-0.89396 - 5.61853I$ |
| $u = 1.09977 - 1.12945I$<br>$a = -0.431917 + 0.251999I$<br>$b = -0.736535 + 0.654115I$ | $6.97511 + 2.09502I$ | $-0.89396 + 1.30967I$ |

**III.** $\boldsymbol{I_3^u = \langle 2a^2 + 2b + 3a + 2,\ 4a^3 + 4a^2 + 5a + 4,\ u + 1\rangle}$

**(i) Arc colorings**

$$a_2 = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} a \\ -a^2 - \frac{3}{2}a - 1 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} a^2 + \frac{5}{2}a + 1 \\ -a^2 - \frac{3}{2}a - 1 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} \frac{3}{2}a^2 - \frac{3}{4}a - 1 \\ -a^2 + \frac{1}{2}a + 1 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} \frac{1}{2}a^2 - \frac{1}{4}a - 2 \\ -a^2 + \frac{1}{2}a + 1 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -2a^2 - a + 1 \\ 2a^2 + a - 1 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -2a^2 - a + 1 \\ 2a^2 + a - 1 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} -2a^2 - 4a - 3 \\ a^2 + \frac{7}{2}a + 2 \end{pmatrix}$$

**(ii) Obstruction class** $= 1$

**(iii) Cusp Shapes** $= \frac{19}{4}a^2 - \frac{11}{8}a + \frac{17}{2}$

**(iv) u-Polynomials at the component**

| Crossings | u-Polynomials at each crossing |
|---|---|
| $c_1, c_2$ | $(u-1)^3$ |
| $c_3, c_7$ | $u^3$ |
| $c_4$ | $(u+1)^3$ |
| $c_5, c_6$ | $u^3 + 2u + 1$ |
| $c_8$ | $u^3 + 3u^2 + 5u + 2$ |
| $c_9, c_{10}, c_{11}$<br>$c_{12}$ | $u^3 + 2u - 1$ |

**(v) Riley Polynomials at the component**

| Crossings | Riley Polynomials at each crossing |
|---|---|
| $c_1, c_2, c_4$ | $(y-1)^3$ |
| $c_3, c_7$ | $y^3$ |
| $c_5, c_6, c_9$<br>$c_{10}, c_{11}, c_{12}$ | $y^3 + 4y^2 + 4y - 1$ |
| $c_8$ | $y^3 + y^2 + 13y - 4$ |

**(vi) Complex Volumes and Cusp Shapes**

| Solutions to $I_3^u$ | $\sqrt{-1}(\mathrm{vol} + \sqrt{-1}CS)$ | Cusp shape |
|---|---|---|
| $u = -1.00000$<br>$a = -0.061957 + 1.066580I$<br>$b = \quad 0.22670 - 1.46771I$ | $-11.08570 - 5.13794I$ | $3.19982 - 2.09434I$ |
| $u = -1.00000$<br>$a = -0.061957 - 1.066580I$<br>$b = \quad 0.22670 + 1.46771I$ | $-11.08570 + 5.13794I$ | $3.19982 + 2.09434I$ |
| $u = -1.00000$<br>$a = -0.876086$<br>$b = -0.453398$ | $-0.857735$ | $13.3500$ |

**IV.**

$$\boldsymbol{I_4^u = \langle u^{12}-3u^{11}+\cdots+b-1,\ -u^{12}+4u^{11}+\cdots+a+1,\ u^{13}-5u^{12}+\cdots+u+1\rangle}$$

**(i) Arc colorings**

$$a_2 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_5 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 1 \\ -u^2 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} u \\ -u^3+u \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} u^{12}-4u^{11}+5u^{10}+2u^9-12u^8+12u^7-3u^6-6u^5+12u^4-7u^3-1 \\ -u^{12}+3u^{11}+\cdots+3u+1 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} 2u^{12}-7u^{11}+\cdots-3u-2 \\ -u^{12}+3u^{11}+\cdots+3u+1 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} u^{12}-6u^{11}+\cdots+3u+1 \\ u^{11}-4u^{10}+5u^9+2u^8-11u^7+8u^6+2u^5-5u^4+4u^3-u^2-u+1 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} -u^{11}+5u^{10}-9u^9+4u^8+9u^7-14u^6+7u^5-u^4-3u^3+5u^2-u+1 \\ -u^3+u^2-1 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -2u^{12}+8u^{11}+\cdots+u+1 \\ 2u^{12}-8u^{11}+\cdots-3u-2 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} -u \\ u \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -2u^{12}+7u^{11}+\cdots+u+1 \\ 2u^{12}-7u^{11}+\cdots-3u-2 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} u^{12}-4u^{11}+\cdots+u^2-2 \\ -u^{12}+3u^{11}+\cdots+4u+1 \end{pmatrix}$$

**(ii) Obstruction class** $= 1$

**(iii) Cusp Shapes**

$= 11u^{11}-46u^{10}+62u^9+11u^8-114u^7+98u^6-4u^5-36u^4+51u^3-26u^2-10u-9$

**(iv) u-Polynomials at the component**

| Crossings | u-Polynomials at each crossing |
|---|---|
| $c_1, c_2$ | $u^{13} + 5u^{12} + \cdots + u - 1$ |
| $c_3$ | $u^{13} - u^{12} + \cdots - u - 1$ |
| $c_4$ | $u^{13} - 5u^{12} + \cdots + u + 1$ |
| $c_5, c_{11}$ | $u^{13} + 5u^{11} + \cdots + 8u - 1$ |
| $c_6, c_9$ | $u^{13} + 5u^{11} + \cdots + 8u + 1$ |
| $c_7$ | $u^{13} + u^{12} + \cdots - u + 1$ |
| $c_8$ | $u^{13} - 2u^{12} + \cdots + 3u + 1$ |
| $c_{10}, c_{12}$ | $u^{13} - 3u^{12} + \cdots - 2u - 1$ |

**(v) Riley Polynomials at the component**

| Crossings | Riley Polynomials at each crossing |
|---|---|
| $c_1, c_2, c_4$ | $y^{13} - 9y^{12} + \cdots - 5y - 1$ |
| $c_3, c_7$ | $y^{13} + 3y^{12} + \cdots - y - 1$ |
| $c_5, c_6, c_9$ $c_{11}$ | $y^{13} + 10y^{12} + \cdots + 62y - 1$ |
| $c_8$ | $y^{13} + 4y^{12} + \cdots + 3y - 1$ |
| $c_{10}, c_{12}$ | $y^{13} - 3y^{12} + \cdots - 4y - 1$ |

**(vi) Complex Volumes and Cusp Shapes**

| Solutions to $I_4^u$ | $\sqrt{-1}(\mathrm{vol}+\sqrt{-1}CS)$ | Cusp shape |
|---|---|---|
| $u=-1.071790+0.063918I$<br>$a=\ 3.89373+0.86609I$<br>$b=-0.350540+1.237350I$ | $-6.72126-1.88681I$ | $0.50306+13.63564I$ |
| $u=-1.071790-0.063918I$<br>$a=\ 3.89373-0.86609I$<br>$b=-0.350540-1.237350I$ | $-6.72126+1.88681I$ | $0.50306-13.63564I$ |
| $u=\ 0.130382+0.815929I$<br>$a=\ 1.43598-0.50475I$<br>$b=\ 0.570608-1.217040I$ | $-3.69895-1.30722I$ | $-2.19900+1.04986I$ |
| $u=\ 0.130382-0.815929I$<br>$a=\ 1.43598+0.50475I$<br>$b=\ 0.570608+1.217040I$ | $-3.69895+1.30722I$ | $-2.19900-1.04986I$ |
| $u=-0.672448$<br>$a=\ 2.97967$<br>$b=\ 0.122783$ | 0.610906 | −22.4950 |
| $u=\ 1.384340+0.198421I$<br>$a=\ 0.111997+1.000200I$<br>$b=\ 0.163145-1.174190I$ | $-9.90727-5.10044I$ | $-4.52290+4.82780I$ |
| $u=\ 1.384340-0.198421I$<br>$a=\ 0.111997-1.000200I$<br>$b=\ 0.163145+1.174190I$ | $-9.90727+5.10044I$ | $-4.52290-4.82780I$ |
| $u=\ 1.44789+0.30492I$<br>$a=-0.265186-0.255350I$<br>$b=\ 0.378653+0.878868I$ | $-8.51258-3.08878I$ | $-7.58668-3.50114I$ |
| $u=\ 1.44789-0.30492I$<br>$a=-0.265186+0.255350I$<br>$b=\ 0.378653-0.878868I$ | $-8.51258+3.08878I$ | $-7.58668+3.50114I$ |
| $u=\ 1.10155+1.08640I$<br>$a=-0.805723-0.406507I$<br>$b=-1.082570+0.295127I$ | $7.14389-4.01026I$ | $-0.32249+2.61344I$ |

| Solutions to $I_4^u$ | $\sqrt{-1}(\mathrm{vol} + \sqrt{-1}CS)$ | Cusp shape |
|---|---|---|
| $u = 1.10155 - 1.08640I$<br>$a = -0.805723 + 0.406507I$<br>$b = -1.082570 - 0.295127I$ | $7.14389 + 4.01026I$ | $-0.32249 - 2.61344I$ |
| $u = -0.156146 + 0.399949I$<br>$a = -1.36064 + 0.71418I$<br>$b = 0.259312 + 1.270730I$ | $-4.92823 + 2.67880I$ | $-0.12459 - 4.50580I$ |
| $u = -0.156146 - 0.399949I$<br>$a = -1.36064 - 0.71418I$<br>$b = 0.259312 - 1.270730I$ | $-4.92823 - 2.67880I$ | $-0.12459 + 4.50580I$ |

## V. $I_5^u = \langle b^2 + ba - a,\ a^2 + a + 1,\ u + 1 \rangle$

**(i) Arc colorings**

$$a_2 = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$
$$a_5 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$
$$a_4 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$
$$a_1 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$
$$a_{10} = \begin{pmatrix} a \\ b \end{pmatrix}$$
$$a_9 = \begin{pmatrix} -b + a \\ b \end{pmatrix}$$
$$a_6 = \begin{pmatrix} -2ba + a + 1 \\ ba - a \end{pmatrix}$$
$$a_{12} = \begin{pmatrix} -ba - 1 \\ ba - a \end{pmatrix}$$
$$a_8 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$
$$a_3 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$
$$a_7 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$
$$a_{11} = \begin{pmatrix} b + 2a \\ -a \end{pmatrix}$$

**(ii) Obstruction class** $= 1$

**(iii) Cusp Shapes** $= -4a - 4$

**(iv) u-Polynomials at the component**

| Crossings | u-Polynomials at each crossing |
|---|---|
| $c_1, c_2$ | $(u-1)^4$ |
| $c_3, c_7$ | $u^4$ |
| $c_4$ | $(u+1)^4$ |
| $c_5, c_6$ | $u^4 - u^3 + 2u^2 - 2u + 1$ |
| $c_8$ | $(u^2 - u + 1)^2$ |
| $c_9, c_{10}, c_{11}$<br>$c_{12}$ | $u^4 + u^3 + 2u^2 + 2u + 1$ |

**(v) Riley Polynomials at the component**

| Crossings | Riley Polynomials at each crossing |
|---|---|
| $c_1, c_2, c_4$ | $(y-1)^4$ |
| $c_3, c_7$ | $y^4$ |
| $c_5, c_6, c_9$<br>$c_{10}, c_{11}, c_{12}$ | $y^4 + 3y^3 + 2y^2 + 1$ |
| $c_8$ | $(y^2 + y + 1)^2$ |

**(vi) Complex Volumes and Cusp Shapes**

| Solutions to $I_5^u$ | $\sqrt{-1}(\mathrm{vol} + \sqrt{-1}CS)$ | Cusp shape |
|---|---|---|
| $u = -1.00000$<br>$a = -0.500000 + 0.866025I$<br>$b = 0.621744 + 0.440597I$ | $-4.93480 + 2.02988I$ | $-2.00000 - 3.46410I$ |
| $u = -1.00000$<br>$a = -0.500000 + 0.866025I$<br>$b = -0.121744 - 1.306620I$ | $-4.93480 + 2.02988I$ | $-2.00000 - 3.46410I$ |
| $u = -1.00000$<br>$a = -0.500000 - 0.866025I$<br>$b = 0.621744 - 0.440597I$ | $-4.93480 - 2.02988I$ | $-2.00000 + 3.46410I$ |
| $u = -1.00000$<br>$a = -0.500000 - 0.866025I$<br>$b = -0.121744 + 1.306620I$ | $-4.93480 - 2.02988I$ | $-2.00000 + 3.46410I$ |

## VI. u-Polynomials

| Crossings | u-Polynomials at each crossing |
|---|---|
| $c_1, c_2$ | $((u-1)^7)(u^5-2u^4+\cdots-u+1)^4(u^{13}+5u^{12}+\cdots+u-1)$ $\cdot(u^{22}-5u^{21}+\cdots+108u+16)$ |
| $c_3$ | $u^7(u^5+u^4+\cdots+2u-2)^4(u^{13}-u^{12}+\cdots-u-1)$ $\cdot(u^{22}-6u^{21}+\cdots+160u-128)$ |
| $c_4$ | $((u+1)^7)(u^5-2u^4+\cdots-u+1)^4(u^{13}-5u^{12}+\cdots+u+1)$ $\cdot(u^{22}-5u^{21}+\cdots+108u+16)$ |
| $c_5$ | $(u^3+2u+1)(u^4-u^3+2u^2-2u+1)(u^{13}+5u^{11}+\cdots+8u-1)$ $\cdot(u^{20}-2u^{19}+\cdots+150u+103)(u^{22}+6u^{20}+\cdots-u-1)$ |
| $c_6$ | $(u^3+2u+1)(u^4-u^3+2u^2-2u+1)(u^{13}+5u^{11}+\cdots+8u+1)$ $\cdot(u^{20}-2u^{19}+\cdots+150u+103)(u^{22}+6u^{20}+\cdots-u-1)$ |
| $c_7$ | $u^7(u^5+u^4+\cdots+2u-2)^4(u^{13}+u^{12}+\cdots-u+1)$ $\cdot(u^{22}-6u^{21}+\cdots+160u-128)$ |
| $c_8$ | $(u^2-u+1)^2(u^2+u+1)^{10}(u^3+3u^2+5u+2)$ $\cdot(u^{13}-2u^{12}+\cdots+3u+1)(u^{22}-14u^{21}+\cdots-464u+32)$ |
| $c_9$ | $(u^3+2u-1)(u^4+u^3+2u^2+2u+1)(u^{13}+5u^{11}+\cdots+8u+1)$ $\cdot(u^{20}-2u^{19}+\cdots+150u+103)(u^{22}+6u^{20}+\cdots-u-1)$ |
| $c_{10}, c_{12}$ | $(u^3+2u-1)(u^4+u^3+2u^2+2u+1)(u^{13}-3u^{12}+\cdots-2u-1)$ $\cdot(u^{20}+2u^{19}+\cdots+13224u+2521)(u^{22}+3u^{21}+\cdots+u+1)$ |
| $c_{11}$ | $(u^3+2u-1)(u^4+u^3+2u^2+2u+1)(u^{13}+5u^{11}+\cdots+8u-1)$ $\cdot(u^{20}-2u^{19}+\cdots+150u+103)(u^{22}+6u^{20}+\cdots-u-1)$ |

## VII. Riley Polynomials

| Crossings | Riley Polynomials at each crossing |
|---|---|
| $c_1, c_2, c_4$ | $((y-1)^7)(y^5+6y^3-y^2-y-1)^4(y^{13}-9y^{12}+\cdots-5y-1)$ $\cdot(y^{22}-11y^{21}+\cdots-10352y+256)$ |
| $c_3, c_7$ | $y^7(y^5+9y^4+\cdots+8y-4)^4(y^{13}+3y^{12}+\cdots-y-1)$ $\cdot(y^{22}+12y^{21}+\cdots-289792y+16384)$ |
| $c_5, c_6, c_9$ $c_{11}$ | $(y^3+4y^2+4y-1)(y^4+3y^3+2y^2+1)(y^{13}+10y^{12}+\cdots+62y-1)$ $\cdot(y^{20}+6y^{19}+\cdots+110988y+10609)(y^{22}+12y^{21}+\cdots-15y+1)$ |
| $c_8$ | $((y^2+y+1)^{12})(y^3+y^2+13y-4)(y^{13}+4y^{12}+\cdots+3y-1)$ $\cdot(y^{22}+6y^{21}+\cdots-36608y+1024)$ |
| $c_{10}, c_{12}$ | $(y^3+4y^2+4y-1)(y^4+3y^3+2y^2+1)(y^{13}-3y^{12}+\cdots-4y-1)$ $\cdot(y^{20}-18y^{19}+\cdots+37696544y+6355441)$ $\cdot(y^{22}-37y^{21}+\cdots-141y+1)$ |