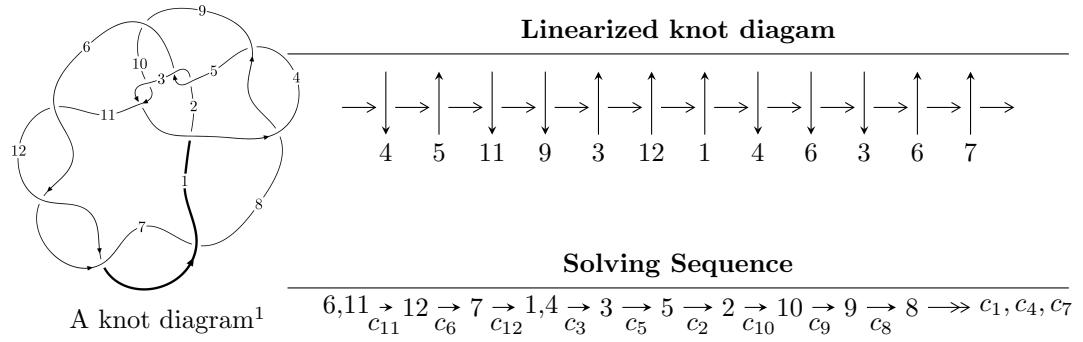


$12n_{0821}$ ($K12n_{0821}$)



Ideals for irreducible components² of X_{par}

$$\begin{aligned}
 I_1^u = & \langle 5u^{12} + 17u^{11} - 8u^{10} - 70u^9 - 18u^8 + 113u^7 + 91u^6 - 46u^5 - 122u^4 - 31u^3 + 55u^2 + 2b + 23u + 12, \\
 & - 11u^{12} - 36u^{11} + \dots + 2a - 27, \\
 & u^{13} + 5u^{12} + 4u^{11} - 16u^{10} - 26u^9 + 15u^8 + 53u^7 + 22u^6 - 36u^5 - 45u^4 - u^3 + 21u^2 + 10u + 4 \rangle \\
 I_2^u = & \langle -u^7 + 5u^5 - u^4 - 7u^3 + 3u^2 + b + 2u - 1, u^7 - 5u^5 + u^4 + 7u^3 - 4u^2 + a - 2u + 3, \\
 & u^8 - 6u^6 + u^5 + 11u^4 - 4u^3 - 6u^2 + 3u + 1 \rangle \\
 I_3^u = & \langle -a^3u^2 - 10a^3u + 2a^2u^2 + 8a^3 - 9a^2u + 16u^2a + 13a^2 - 14au + 19u^2 + 29b - 12a - 13u - 36, \\
 & - 2a^3u^2 + a^4 + a^3u + 3a^2u^2 + 4a^3 - a^2u + 12u^2a - 5a^2 - 7au - 27a + u + 1, u^3 - u^2 - 2u + 1 \rangle
 \end{aligned}$$

* 3 irreducible components of $\dim_{\mathbb{C}} = 0$, with total 33 representations.

¹The image of knot diagram is generated by the software “**Draw programme**” developed by Andrew Bartholomew(<http://www.layer8.co.uk/math/draw/index.htm#Running-draw>), where we modified some parts for our purpose(<https://github.com/CATsTAILs/LinksPainter>).

²All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

$$\text{I. } I_1^u = \langle 5u^{12} + 17u^{11} + \cdots + 2b + 12, -11u^{12} - 36u^{11} + \cdots + 2a - 27, u^{13} + 5u^{12} + \cdots + 10u + 4 \rangle$$

(i) Arc colorings

$$\begin{aligned} a_6 &= \begin{pmatrix} 0 \\ u \end{pmatrix} \\ a_{11} &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ a_{12} &= \begin{pmatrix} 1 \\ -u^2 \end{pmatrix} \\ a_7 &= \begin{pmatrix} u \\ -u^3 + u \end{pmatrix} \\ a_1 &= \begin{pmatrix} -u^2 + 1 \\ u^4 - 2u^2 \end{pmatrix} \\ a_4 &= \begin{pmatrix} \frac{11}{2}u^{12} + 18u^{11} + \cdots + 24u + \frac{27}{2} \\ -\frac{5}{2}u^{12} - \frac{17}{2}u^{11} + \cdots - \frac{23}{2}u - 6 \end{pmatrix} \\ a_3 &= \begin{pmatrix} 3u^{12} + \frac{19}{2}u^{11} + \cdots + \frac{25}{2}u + \frac{15}{2} \\ -\frac{5}{2}u^{12} - \frac{17}{2}u^{11} + \cdots - \frac{23}{2}u - 6 \end{pmatrix} \\ a_5 &= \begin{pmatrix} \frac{11}{4}u^{12} + \frac{37}{4}u^{11} + \cdots + \frac{49}{4}u + 7 \\ -\frac{1}{2}u^{12} - \frac{3}{2}u^{11} + \cdots - \frac{3}{2}u - 1 \end{pmatrix} \\ a_2 &= \begin{pmatrix} -\frac{11}{4}u^{12} - \frac{37}{4}u^{11} + \cdots - \frac{53}{4}u - 6 \\ -\frac{1}{2}u^{12} - \frac{3}{2}u^{11} + \cdots - \frac{3}{2}u - 1 \end{pmatrix} \\ a_{10} &= \begin{pmatrix} \frac{1}{4}u^{12} + \frac{3}{4}u^{11} + \cdots - \frac{1}{4}u + 1 \\ \frac{1}{2}u^{12} + \frac{3}{2}u^{11} + \cdots + \frac{3}{2}u + 1 \end{pmatrix} \\ a_9 &= \begin{pmatrix} \frac{1}{4}u^{12} + \frac{3}{4}u^{11} + \cdots - \frac{1}{4}u + 1 \\ -\frac{1}{2}u^{12} - \frac{3}{2}u^{11} + \cdots - \frac{5}{2}u - 1 \end{pmatrix} \\ a_8 &= \begin{pmatrix} u^3 - 2u \\ -u^5 + 3u^3 - u \end{pmatrix} \end{aligned}$$

(ii) Obstruction class = -1

$$\text{(iii) Cusp Shapes} = -17u^{12} - 55u^{11} + 30u^{10} + 220u^9 + 44u^8 - 346u^7 - 267u^6 + 135u^5 + 358u^4 + 81u^3 - 157u^2 - 56u - 34$$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
c_1	$u^{13} - u^{12} + \cdots + 15u + 1$
c_2, c_5	$u^{13} + 6u^{12} + \cdots + 12u + 8$
c_3, c_4, c_8 c_{10}	$u^{13} - u^{12} + \cdots + u - 1$
c_6, c_7, c_{11} c_{12}	$u^{13} - 5u^{12} + \cdots + 10u - 4$
c_9	$u^{13} + 15u^{11} + \cdots - 15u^2 - 1$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
c_1	$y^{13} + 39y^{12} + \cdots + 155y - 1$
c_2, c_5	$y^{13} - 14y^{12} + \cdots + 80y - 64$
c_3, c_4, c_8 c_{10}	$y^{13} + 7y^{12} + \cdots - y - 1$
c_6, c_7, c_{11} c_{12}	$y^{13} - 17y^{12} + \cdots - 68y - 16$
c_9	$y^{13} + 30y^{12} + \cdots - 30y - 1$

(vi) Complex Volumes and Cusp Shapes

Solutions to I_1^u	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.497615 + 0.876393I$		
$a = -0.520432 - 0.143375I$	$5.56724 - 2.86079I$	$6.58762 + 4.73580I$
$b = -0.286884 - 1.048300I$		
$u = -0.497615 - 0.876393I$		
$a = -0.520432 + 0.143375I$	$5.56724 + 2.86079I$	$6.58762 - 4.73580I$
$b = -0.286884 + 1.048300I$		
$u = 0.977918 + 0.258584I$		
$a = -0.50072 - 1.42125I$	$3.49671 + 3.30133I$	$5.66986 - 7.29619I$
$b = -0.432880 + 0.770070I$		
$u = 0.977918 - 0.258584I$		
$a = -0.50072 + 1.42125I$	$3.49671 - 3.30133I$	$5.66986 + 7.29619I$
$b = -0.432880 - 0.770070I$		
$u = -1.15240$		
$a = -0.0663024$	2.44636	4.36790
$b = -0.413299$		
$u = 1.276260 + 0.459752I$		
$a = 0.350061 + 1.031200I$	$11.16010 + 7.44705I$	$5.81652 - 5.20775I$
$b = 0.82833 - 1.24672I$		
$u = 1.276260 - 0.459752I$		
$a = 0.350061 - 1.031200I$	$11.16010 - 7.44705I$	$5.81652 + 5.20775I$
$b = 0.82833 + 1.24672I$		
$u = -0.158213 + 0.403429I$		
$a = 0.826836 - 0.260199I$	$-0.016018 - 0.973727I$	$-0.21220 + 6.98709I$
$b = 0.295622 + 0.443698I$		
$u = -0.158213 - 0.403429I$		
$a = 0.826836 + 0.260199I$	$-0.016018 + 0.973727I$	$-0.21220 - 6.98709I$
$b = 0.295622 - 0.443698I$		
$u = -1.71570 + 0.06763I$		
$a = 0.12881 - 1.62879I$	$13.07970 - 4.61256I$	$6.03428 + 7.03944I$
$b = 0.519770 + 0.949390I$		

Solutions to I_1^u	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -1.71570 - 0.06763I$		
$a = 0.12881 + 1.62879I$	$13.07970 + 4.61256I$	$6.03428 - 7.03944I$
$b = 0.519770 - 0.949390I$		
$u = -1.80645 + 0.12280I$		
$a = 0.24859 + 1.52769I$	$-17.2391 - 10.0928I$	$5.92000 + 4.03274I$
$b = -1.21731 - 1.42694I$		
$u = -1.80645 - 0.12280I$		
$a = 0.24859 - 1.52769I$	$-17.2391 + 10.0928I$	$5.92000 - 4.03274I$
$b = -1.21731 + 1.42694I$		

$$\text{II. } I_2^u = \langle -u^7 + 5u^5 - u^4 - 7u^3 + 3u^2 + b + 2u - 1, u^7 - 5u^5 + u^4 + 7u^3 - 4u^2 + a - 2u + 3, u^8 - 6u^6 + u^5 + 11u^4 - 4u^3 - 6u^2 + 3u + 1 \rangle$$

(i) **Arc colorings**

$$\begin{aligned} a_6 &= \begin{pmatrix} 0 \\ u \end{pmatrix} \\ a_{11} &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ a_{12} &= \begin{pmatrix} 1 \\ -u^2 \end{pmatrix} \\ a_7 &= \begin{pmatrix} u \\ -u^3 + u \end{pmatrix} \\ a_1 &= \begin{pmatrix} -u^2 + 1 \\ u^4 - 2u^2 \end{pmatrix} \\ a_4 &= \begin{pmatrix} -u^7 + 5u^5 - u^4 - 7u^3 + 4u^2 + 2u - 3 \\ u^7 - 5u^5 + u^4 + 7u^3 - 3u^2 - 2u + 1 \end{pmatrix} \\ a_3 &= \begin{pmatrix} u^2 - 2 \\ u^7 - 5u^5 + u^4 + 7u^3 - 3u^2 - 2u + 1 \end{pmatrix} \\ a_5 &= \begin{pmatrix} -u^5 + 4u^3 - 4u \\ -u^4 + 3u^2 - 1 \end{pmatrix} \\ a_2 &= \begin{pmatrix} u^5 - u^4 - 4u^3 + 3u^2 + 3u - 1 \\ u^4 - 3u^2 + 1 \end{pmatrix} \\ a_{10} &= \begin{pmatrix} u^7 - 6u^5 + u^4 + 10u^3 - 4u^2 - 3u + 3 \\ -u^3 + 2u - 1 \end{pmatrix} \\ a_9 &= \begin{pmatrix} u^7 - 6u^5 + u^4 + 10u^3 - 4u^2 - 3u + 3 \\ u^5 - 4u^3 + 3u - 1 \end{pmatrix} \\ a_8 &= \begin{pmatrix} -u^3 + 2u \\ u^5 - 3u^3 + u \end{pmatrix} \end{aligned}$$

(ii) **Obstruction class** = 1

(iii) **Cusp Shapes** = $-3u^7 + 2u^6 + 21u^5 - 12u^4 - 42u^3 + 24u^2 + 17u - 10$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
c_1	$u^8 + 3u^7 + 6u^6 + 4u^5 - 7u^4 - 12u^3 - u^2 + 6u + 1$
c_2	$u^8 + 3u^7 - u^6 - 11u^5 - 6u^4 + 11u^3 + 8u^2 - 3u - 1$
c_3, c_8	$u^8 + u^7 - 2u^6 - 2u^5 - u^4 + u^2 + 1$
c_4, c_{10}	$u^8 - u^7 - 2u^6 + 2u^5 - u^4 + u^2 + 1$
c_5	$u^8 - 3u^7 - u^6 + 11u^5 - 6u^4 - 11u^3 + 8u^2 + 3u - 1$
c_6, c_7	$u^8 - 6u^6 - u^5 + 11u^4 + 4u^3 - 6u^2 - 3u + 1$
c_9	$u^8 + u^6 - u^4 - 2u^3 - 2u^2 + u + 1$
c_{11}, c_{12}	$u^8 - 6u^6 + u^5 + 11u^4 - 4u^3 - 6u^2 + 3u + 1$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
c_1	$y^8 + 3y^7 - 2y^6 - 30y^5 + 99y^4 - 166y^3 + 131y^2 - 38y + 1$
c_2, c_5	$y^8 - 11y^7 + 55y^6 - 159y^5 + 278y^4 - 281y^3 + 142y^2 - 25y + 1$
c_3, c_4, c_8 c_{10}	$y^8 - 5y^7 + 6y^6 + 2y^5 - y^4 - 6y^3 - y^2 + 2y + 1$
c_6, c_7, c_{11} c_{12}	$y^8 - 12y^7 + 58y^6 - 145y^5 + 203y^4 - 166y^3 + 82y^2 - 21y + 1$
c_9	$y^8 + 2y^7 - y^6 - 6y^5 - y^4 + 2y^3 + 6y^2 - 5y + 1$

(vi) Complex Volumes and Cusp Shapes

Solutions to I_2^u	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.868162$		
$a = 0.196614$	-1.71749	5.61020
$b = -1.44291$		
$u = 0.733070 + 0.412657I$		
$a = -1.46575 - 0.08392I$	4.59844 + 1.46844I	5.92040 - 3.51787I
$b = -0.167149 + 0.688931I$		
$u = 0.733070 - 0.412657I$		
$a = -1.46575 + 0.08392I$	4.59844 - 1.46844I	5.92040 + 3.51787I
$b = -0.167149 - 0.688931I$		
$u = 1.35093$		
$a = 0.698866$	1.54653	-4.73980
$b = -0.873848$		
$u = 1.69498$		
$a = -0.701727$	7.46249	4.83590
$b = 1.57470$		
$u = -1.69932 + 0.10356I$		
$a = 0.446197 - 1.151580I$	13.38720 - 3.48023I	8.31022 + 1.19329I
$b = 0.430778 + 0.799616I$		
$u = -1.69932 - 0.10356I$		
$a = 0.446197 + 1.151580I$	13.38720 + 3.48023I	8.31022 - 1.19329I
$b = 0.430778 - 0.799616I$		
$u = -0.245247$		
$a = -3.15466$	-3.78433	-12.1680
$b = 1.21480$		

$$\text{III. } I_3^u = \langle -a^3u^2 + 2a^2u^2 + \dots - 12a - 36, -2a^3u^2 + 3a^2u^2 + \dots - 27a + 1, u^3 - u^2 - 2u + 1 \rangle$$

(i) Arc colorings

$$a_6 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} 1 \\ -u^2 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} u \\ -u^2 - u + 1 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -u^2 + 1 \\ u^2 + u - 1 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} a \\ 0.0344828a^3u^2 - 0.0689655a^2u^2 + \dots + 0.413793a + 1.24138 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} 0.0344828a^3u^2 - 0.0689655a^2u^2 + \dots + 1.41379a + 1.24138 \\ 0.0344828a^3u^2 - 0.0689655a^2u^2 + \dots + 0.413793a + 1.24138 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} 0.0344828a^3u^2 - 0.0689655a^2u^2 + \dots + 1.41379a - 0.758621 \\ 0.379310a^3u^2 - 0.758621a^2u^2 + \dots + 0.551724a - 0.344828 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} -0.0344828a^3u^2 + 0.0689655a^2u^2 + \dots - 1.41379a + 0.758621 \\ -0.206897a^3u^2 + 0.413793a^2u^2 + \dots - 0.482759a + 0.551724 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} 0.0344828a^3u^2 - 0.0689655a^2u^2 + \dots + 1.41379a - 0.758621 \\ -0.206897a^3u^2 - 0.586207a^2u^2 + \dots - 0.482759a - 1.44828 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} 0.0344828a^3u^2 - 0.0689655a^2u^2 + \dots + 1.41379a - 0.758621 \\ -0.379310a^3u^2 - 0.241379a^2u^2 + \dots - 0.551724a - 1.65517 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} u^2 - 1 \\ -u^2 \end{pmatrix}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes = 6

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
c_1	$u^{12} - u^{11} + \cdots - 42u - 1$
c_2, c_5	$(u^2 - u - 1)^6$
c_3, c_4, c_8 c_{10}	$u^{12} - u^{11} + u^9 + 8u^8 + u^7 - 7u^6 - 3u^5 - 6u^4 + 10u^3 - 18u^2 + 12u + 1$
c_6, c_7, c_{11} c_{12}	$(u^3 + u^2 - 2u - 1)^4$
c_9	$u^{12} + u^{11} + \cdots + 84u - 29$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
c_1	$y^{12} + 15y^{11} + \dots - 1900y + 1$
c_2, c_5	$(y^2 - 3y + 1)^6$
c_3, c_4, c_8 c_{10}	$y^{12} - y^{11} + \dots - 180y + 1$
c_6, c_7, c_{11} c_{12}	$(y^3 - 5y^2 + 6y - 1)^4$
c_9	$y^{12} + 11y^{11} + \dots - 7288y + 841$

(vi) Complex Volumes and Cusp Shapes

Solutions to I_3^u	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -1.24698$		
$a = 0.288735 + 1.074830I$	10.2926	6.00000
$b = 1.00883 - 1.07483I$		
$u = -1.24698$		
$a = 0.288735 - 1.074830I$	10.2926	6.00000
$b = 1.00883 + 1.07483I$		
$u = -1.24698$		
$a = -0.570245$	2.39690	6.00000
$b = 0.0746199$		
$u = -1.24698$		
$a = 0.349671$	2.39690	6.00000
$b = -0.845296$		
$u = 0.445042$		
$a = 0.0516489$	-3.24287	6.00000
$b = 1.33706$		
$u = 0.445042$		
$a = 2.45072$	-3.24287	6.00000
$b = -1.06201$		
$u = 0.445042$		
$a = -3.27564 + 0.82853I$	4.65281	6.00000
$b = -0.360046 - 0.828531I$		
$u = 0.445042$		
$a = -3.27564 - 0.82853I$	4.65281	6.00000
$b = -0.360046 + 0.828531I$		
$u = 1.80194$		
$a = -0.213846 + 1.148430I$	13.6765	6.00000
$b = 0.556829 - 1.148430I$		
$u = 1.80194$		
$a = -0.213846 - 1.148430I$	13.6765	6.00000
$b = 0.556829 + 1.148430I$		

Solutions to I_3^u	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 1.80194$		
$a = 0.55986 + 1.31903I$	-17.9063	6.00000
$b = -1.45780 - 1.31903I$		
$u = 1.80194$		
$a = 0.55986 - 1.31903I$	-17.9063	6.00000
$b = -1.45780 + 1.31903I$		

IV. u-Polynomials

Crossings	u-Polynomials at each crossing
c_1	$(u^8 + 3u^7 + 6u^6 + 4u^5 - 7u^4 - 12u^3 - u^2 + 6u + 1)$ $\cdot (u^{12} - u^{11} + \dots - 42u - 1)(u^{13} - u^{12} + \dots + 15u + 1)$
c_2	$(u^2 - u - 1)^6(u^8 + 3u^7 - u^6 - 11u^5 - 6u^4 + 11u^3 + 8u^2 - 3u - 1)$ $\cdot (u^{13} + 6u^{12} + \dots + 12u + 8)$
c_3, c_8	$(u^8 + u^7 - 2u^6 - 2u^5 - u^4 + u^2 + 1)$ $\cdot (u^{12} - u^{11} + u^9 + 8u^8 + u^7 - 7u^6 - 3u^5 - 6u^4 + 10u^3 - 18u^2 + 12u + 1)$ $\cdot (u^{13} - u^{12} + \dots + u - 1)$
c_4, c_{10}	$(u^8 - u^7 - 2u^6 + 2u^5 - u^4 + u^2 + 1)$ $\cdot (u^{12} - u^{11} + u^9 + 8u^8 + u^7 - 7u^6 - 3u^5 - 6u^4 + 10u^3 - 18u^2 + 12u + 1)$ $\cdot (u^{13} - u^{12} + \dots + u - 1)$
c_5	$(u^2 - u - 1)^6(u^8 - 3u^7 - u^6 + 11u^5 - 6u^4 - 11u^3 + 8u^2 + 3u - 1)$ $\cdot (u^{13} + 6u^{12} + \dots + 12u + 8)$
c_6, c_7	$(u^3 + u^2 - 2u - 1)^4(u^8 - 6u^6 - u^5 + 11u^4 + 4u^3 - 6u^2 - 3u + 1)$ $\cdot (u^{13} - 5u^{12} + \dots + 10u - 4)$
c_9	$(u^8 + u^6 - u^4 - 2u^3 - 2u^2 + u + 1)(u^{12} + u^{11} + \dots + 84u - 29)$ $\cdot (u^{13} + 15u^{11} + \dots - 15u^2 - 1)$
c_{11}, c_{12}	$(u^3 + u^2 - 2u - 1)^4(u^8 - 6u^6 + u^5 + 11u^4 - 4u^3 - 6u^2 + 3u + 1)$ $\cdot (u^{13} - 5u^{12} + \dots + 10u - 4)$

V. Riley Polynomials

Crossings	Riley Polynomials at each crossing
c_1	$(y^8 + 3y^7 - 2y^6 - 30y^5 + 99y^4 - 166y^3 + 131y^2 - 38y + 1)$ $\cdot (y^{12} + 15y^{11} + \dots - 1900y + 1)(y^{13} + 39y^{12} + \dots + 155y - 1)$
c_2, c_5	$(y^2 - 3y + 1)^6$ $\cdot (y^8 - 11y^7 + 55y^6 - 159y^5 + 278y^4 - 281y^3 + 142y^2 - 25y + 1)$ $\cdot (y^{13} - 14y^{12} + \dots + 80y - 64)$
c_3, c_4, c_8 c_{10}	$(y^8 - 5y^7 + 6y^6 + 2y^5 - y^4 - 6y^3 - y^2 + 2y + 1)$ $\cdot (y^{12} - y^{11} + \dots - 180y + 1)(y^{13} + 7y^{12} + \dots - y - 1)$
c_6, c_7, c_{11} c_{12}	$(y^3 - 5y^2 + 6y - 1)^4$ $\cdot (y^8 - 12y^7 + 58y^6 - 145y^5 + 203y^4 - 166y^3 + 82y^2 - 21y + 1)$ $\cdot (y^{13} - 17y^{12} + \dots - 68y - 16)$
c_9	$(y^8 + 2y^7 - y^6 - 6y^5 - y^4 + 2y^3 + 6y^2 - 5y + 1)$ $\cdot (y^{12} + 11y^{11} + \dots - 7288y + 841)(y^{13} + 30y^{12} + \dots - 30y - 1)$