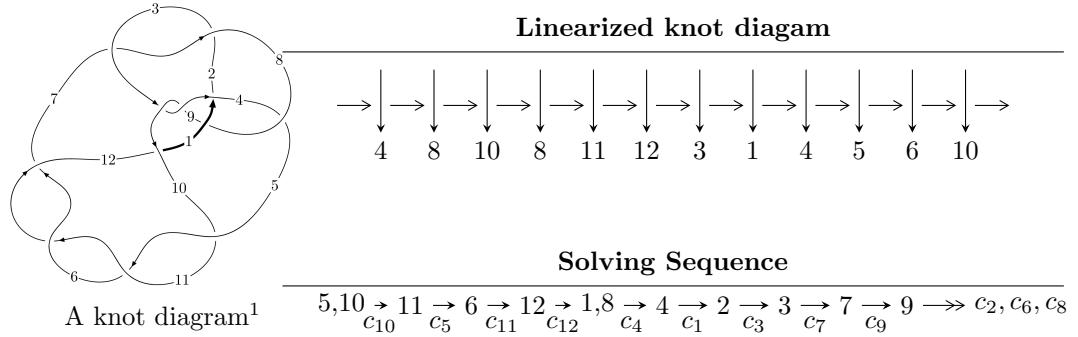


$12n_{0850}$  ( $K12n_{0850}$ )



### Ideals for irreducible components<sup>2</sup> of $X_{\text{par}}$

$$\begin{aligned}
 I_1^u &= \langle -u^{10} - 5u^9 - 4u^8 + 8u^7 - u^6 - 16u^5 + 19u^4 + 20u^3 - 15u^2 + 2b + 5u + 4, \\
 &\quad - 3u^{10} - 10u^9 - u^8 + 14u^7 - 17u^6 - 17u^5 + 47u^4 + 13u^3 - 25u^2 + 2a + 18u + 7, \\
 &\quad u^{11} + 5u^{10} + 6u^9 - 4u^8 - 3u^7 + 14u^6 - 5u^5 - 30u^4 - u^3 + 9u^2 - 10u - 4 \rangle \\
 I_2^u &= \langle -u^5 + 4u^3 + b - 3u, u^5 - 5u^3 + u^2 + a + 6u - 3, u^7 - u^6 - 5u^5 + 5u^4 + 6u^3 - 7u^2 + u + 1 \rangle \\
 I_3^u &= \langle b, a + 1, u + 1 \rangle \\
 I_4^u &= \langle -a^3 + b + a + 1, a^4 + a^3 - 2a - 1, u - 1 \rangle
 \end{aligned}$$

\* 4 irreducible components of  $\dim_{\mathbb{C}} = 0$ , with total 23 representations.

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<sup>1</sup>The image of knot diagram is generated by the software “**Draw programme**” developed by Andrew Bartholomew(<http://www.layer8.co.uk/math/draw/index.htm#Running-draw>), where we modified some parts for our purpose(<https://github.com/CATsTAILs/LinksPainter>).

<sup>2</sup>All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

$$\text{I. } I_1^u = \langle -u^{10} - 5u^9 + \dots + 2b + 4, -3u^{10} - 10u^9 + \dots + 2a + 7, u^{11} + 5u^{10} + \dots - 10u - 4 \rangle$$

(i) Arc colorings

$$a_5 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 1 \\ u^2 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -u \\ -u^3 + u \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} -u^2 + 1 \\ -u^4 + 2u^2 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} u^4 - 3u^2 + 1 \\ -u^4 + 2u^2 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} \frac{3}{2}u^{10} + 5u^9 + \dots - 9u - \frac{7}{2} \\ \frac{1}{2}u^{10} + \frac{5}{2}u^9 + \dots - \frac{5}{2}u - 2 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} \frac{1}{4}u^{10} + \frac{3}{4}u^9 + \dots - \frac{5}{4}u - 1 \\ -\frac{1}{2}u^{10} - \frac{3}{2}u^9 + \dots + \frac{5}{2}u + 1 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} -\frac{1}{2}u^9 - \frac{1}{2}u^8 + \dots + \frac{1}{2}u + \frac{3}{2} \\ \frac{5}{2}u^{10} + \frac{17}{2}u^9 + \dots - \frac{23}{2}u - 6 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} -\frac{1}{4}u^{10} - \frac{3}{4}u^9 + \dots - \frac{3}{4}u^2 + \frac{5}{4}u \\ -\frac{1}{2}u^{10} - \frac{3}{2}u^9 + \dots + \frac{5}{2}u + 1 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} u^3 - 2u \\ u^5 - 3u^3 + u \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -u^{10} - \frac{5}{2}u^9 + \dots + \frac{7}{2}u + \frac{5}{2} \\ -\frac{1}{2}u^{10} - \frac{5}{2}u^9 + \dots + \frac{7}{2}u + 2 \end{pmatrix}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes

$$= -7u^{10} - 25u^9 - 6u^8 + 36u^7 - 35u^6 - 50u^5 + 113u^4 + 43u^3 - 65u^2 + 46u + 2$$

**(iv) u-Polynomials at the component**

Crossings	u-Polynomials at each crossing
$c_1$	$u^{11} - 7u^{10} + \cdots - 2u + 2$
$c_2, c_3, c_7$ $c_9$	$u^{11} + 14u^9 + 4u^8 + 46u^7 + 67u^6 - 66u^5 - 7u^4 - 30u^3 - 9u^2 - 3u - 1$
$c_4, c_8$	$u^{11} + u^{10} + \cdots - 4u - 1$
$c_5, c_6, c_{10}$ $c_{11}$	$u^{11} + 5u^{10} + \cdots - 10u - 4$
$c_{12}$	$u^{11} - 3u^{10} + \cdots - 3192u - 576$

**(v) Riley Polynomials at the component**

Crossings	Riley Polynomials at each crossing
$c_1$	$y^{11} - 31y^{10} + \cdots + 200y - 4$
$c_2, c_3, c_7$ $c_9$	$y^{11} + 28y^{10} + \cdots - 9y - 1$
$c_4, c_8$	$y^{11} + 17y^{10} + \cdots + 24y - 1$
$c_5, c_6, c_{10}$ $c_{11}$	$y^{11} - 13y^{10} + \cdots + 172y - 16$
$c_{12}$	$y^{11} + 67y^{10} + \cdots + 7508160y - 331776$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 1.18814$		
$a = 0.348347$	-5.45154	-15.3520
$b = -0.566087$		
$u = 0.651462 + 1.063750I$		
$a = 1.81578 + 0.65642I$	$12.65630 - 3.45618I$	$-10.65599 + 2.10885I$
$b = -1.77237 + 0.06609I$		
$u = 0.651462 - 1.063750I$		
$a = 1.81578 - 0.65642I$	$12.65630 + 3.45618I$	$-10.65599 - 2.10885I$
$b = -1.77237 - 0.06609I$		
$u = 0.496165 + 0.539201I$		
$a = -1.042860 - 0.661841I$	$2.08534 - 1.86536I$	$-10.69284 + 5.33447I$
$b = 1.138470 - 0.357731I$		
$u = 0.496165 - 0.539201I$		
$a = -1.042860 + 0.661841I$	$2.08534 + 1.86536I$	$-10.69284 - 5.33447I$
$b = 1.138470 + 0.357731I$		
$u = -1.53157 + 0.15203I$		
$a = -0.354779 + 0.587363I$	$-4.66784 + 4.30939I$	$-14.0390 - 6.9085I$
$b = 1.12977 + 1.06011I$		
$u = -1.53157 - 0.15203I$		
$a = -0.354779 - 0.587363I$	$-4.66784 - 4.30939I$	$-14.0390 + 6.9085I$
$b = 1.12977 - 1.06011I$		
$u = -0.330126$		
$a = 0.768686$	-0.487897	-20.3390
$b = -0.139205$		
$u = -1.64929 + 0.39522I$		
$a = 0.914271 - 0.873133I$	$5.23140 + 8.93346I$	$-13.21655 - 3.59394I$
$b = -1.77328 - 0.21395I$		
$u = -1.64929 - 0.39522I$		
$a = 0.914271 + 0.873133I$	$5.23140 - 8.93346I$	$-13.21655 + 3.59394I$
$b = -1.77328 + 0.21395I$		

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -1.79155$		
$a = 0.218139$	-16.4464	-7.09980
$b = -0.739878$		

$$\text{II. } I_2^u = \langle -u^5 + 4u^3 + b - 3u, u^5 - 5u^3 + u^2 + a + 6u - 3, u^7 - u^6 - 5u^5 + 5u^4 + 6u^3 - 7u^2 + u + 1 \rangle$$

(i) **Arc colorings**

$$\begin{aligned}
a_5 &= \begin{pmatrix} 0 \\ u \end{pmatrix} \\
a_{10} &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\
a_{11} &= \begin{pmatrix} 1 \\ u^2 \end{pmatrix} \\
a_6 &= \begin{pmatrix} -u \\ -u^3 + u \end{pmatrix} \\
a_{12} &= \begin{pmatrix} -u^2 + 1 \\ -u^4 + 2u^2 \end{pmatrix} \\
a_1 &= \begin{pmatrix} u^4 - 3u^2 + 1 \\ -u^4 + 2u^2 \end{pmatrix} \\
a_8 &= \begin{pmatrix} -u^5 + 5u^3 - u^2 - 6u + 3 \\ u^5 - 4u^3 + 3u \end{pmatrix} \\
a_4 &= \begin{pmatrix} -u^6 + u^5 + 4u^4 - 5u^3 - 2u^2 + 7u - 4 \\ u^4 - 3u^2 + 1 \end{pmatrix} \\
a_2 &= \begin{pmatrix} -2u^6 + 2u^5 + 10u^4 - 10u^3 - 11u^2 + 14u - 4 \\ u^6 - 5u^4 + u^3 + 6u^2 - 3u \end{pmatrix} \\
a_3 &= \begin{pmatrix} -u^6 + u^5 + 5u^4 - 5u^3 - 5u^2 + 7u - 3 \\ u^4 - 3u^2 + 1 \end{pmatrix} \\
a_7 &= \begin{pmatrix} u^3 - 2u \\ u^5 - 3u^3 + u \end{pmatrix} \\
a_9 &= \begin{pmatrix} -u^5 + 5u^3 - u^2 - 6u + 4 \\ -u^3 + 2u \end{pmatrix}
\end{aligned}$$

(ii) **Obstruction class = 1**

(iii) **Cusp Shapes** =  $2u^6 - 2u^5 - 12u^4 + 5u^3 + 16u^2 - 3u - 10$

**(iv) u-Polynomials at the component**

Crossings	u-Polynomials at each crossing
$c_1$	$u^7 - 6u^6 + 12u^5 - 11u^4 + 7u^3 - 3u^2 - 1$
$c_2, c_9$	$u^7 - u^6 + u^5 - 2u^4 - u^2 + 1$
$c_3, c_7$	$u^7 + u^6 + u^5 + 2u^4 + u^2 - 1$
$c_4, c_8$	$u^7 - u^5 - 2u^3 + u^2 - u + 1$
$c_5, c_6$	$u^7 + u^6 - 5u^5 - 5u^4 + 6u^3 + 7u^2 + u - 1$
$c_{10}, c_{11}$	$u^7 - u^6 - 5u^5 + 5u^4 + 6u^3 - 7u^2 + u + 1$
$c_{12}$	$u^7 - 5u^6 + 7u^5 - 5u^4 - 4u^3 - 7u^2 + 5u + 1$

**(v) Riley Polynomials at the component**

Crossings	Riley Polynomials at each crossing
$c_1$	$y^7 - 12y^6 + 26y^5 + 11y^4 - 29y^3 - 31y^2 - 6y - 1$
$c_2, c_3, c_7$ $c_9$	$y^7 + y^6 - 3y^5 - 6y^4 - 2y^3 + 3y^2 + 2y - 1$
$c_4, c_8$	$y^7 - 2y^6 - 3y^5 + 2y^4 + 6y^3 + 3y^2 - y - 1$
$c_5, c_6, c_{10}$ $c_{11}$	$y^7 - 11y^6 + 47y^5 - 97y^4 + 98y^3 - 47y^2 + 15y - 1$
$c_{12}$	$y^7 - 11y^6 - 9y^5 - 141y^4 + 26y^3 - 79y^2 + 39y - 1$

**(vi) Complex Volumes and Cusp Shapes**

Solutions to $I_2^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.602602 + 0.366097I$		
$a = -0.802378 - 0.958802I$	$3.42389 - 1.23175I$	$-6.47743 + 5.11160I$
$b = 1.74200 - 0.23095I$		
$u = 0.602602 - 0.366097I$		
$a = -0.802378 + 0.958802I$	$3.42389 + 1.23175I$	$-6.47743 - 5.11160I$
$b = 1.74200 + 0.23095I$		
$u = 1.50894$		
$a = 1.02523$	$-10.4234$	$-15.1670$
$b = -1.39324$		
$u = -1.59539 + 0.14916I$		
$a = -0.285687 + 0.511963I$	$-4.17528 + 3.26775I$	$-10.72162 - 1.02180I$
$b = 1.59460 + 0.65214I$		
$u = -1.59539 - 0.14916I$		
$a = -0.285687 - 0.511963I$	$-4.17528 - 3.26775I$	$-10.72162 + 1.02180I$
$b = 1.59460 - 0.65214I$		
$u = -0.293328$		
$a = 4.54991$	$-4.14361$	$-7.95280$
$b = -0.781203$		
$u = 1.76997$		
$a = -0.399006$	$-16.8289$	$-28.4820$
$b = 0.501243$		

$$\text{III. } I_3^u = \langle b, a+1, u+1 \rangle$$

(i) Arc colorings

$$a_5 = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} -3 \\ -1 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} -2 \\ -1 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes = -24

**(iv) u-Polynomials at the component**

Crossings	u-Polynomials at each crossing
$c_1$	$u - 2$
$c_2, c_4, c_8$ $c_9, c_{10}, c_{11}$ $c_{12}$	$u + 1$
$c_3, c_5, c_6$ $c_7$	$u - 1$

**(v) Riley Polynomials at the component**

Crossings	Riley Polynomials at each crossing
$c_1$	$y - 4$
$c_2, c_3, c_4$	
$c_5, c_6, c_7$	
$c_8, c_9, c_{10}$	$y - 1$
$c_{11}, c_{12}$	

**(vi) Complex Volumes and Cusp Shapes**

Solutions to $I_3^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -1.00000$		
$a = -1.00000$	-6.57974	-24.0000
$b = 0$		

$$\text{IV. } I_4^u = \langle -a^3 + b + a + 1, a^4 + a^3 - 2a - 1, u - 1 \rangle$$

(i) Arc colorings

$$a_5 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} a \\ a^3 - a - 1 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} a^2 \\ -a^3 - a^2 + a + 2 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} -a - 2 \\ a^3 - 1 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} -a^3 + a + 2 \\ -a^3 - a^2 + a + 2 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -a^3 + a + 1 \\ 2a^3 - a - 2 \end{pmatrix}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes = -14

**(iv) u-Polynomials at the component**

Crossings	u-Polynomials at each crossing
$c_1$	$(u^2 + u - 1)^2$
$c_2, c_3, c_7$ $c_9$	$u^4 - u^3 + 2u^2 - 4u + 1$
$c_4, c_8$	$u^4 + u^3 - 2u - 1$
$c_5, c_6, c_{10}$ $c_{11}$	$(u - 1)^4$
$c_{12}$	$(u + 1)^4$

**(v) Riley Polynomials at the component**

Crossings	Riley Polynomials at each crossing
$c_1$	$(y^2 - 3y + 1)^2$
$c_2, c_3, c_7$ $c_9$	$y^4 + 3y^3 - 2y^2 - 12y + 1$
$c_4, c_8$	$y^4 - y^3 + 2y^2 - 4y + 1$
$c_5, c_6, c_{10}$ $c_{11}, c_{12}$	$(y - 1)^4$

**(vi) Complex Volumes and Cusp Shapes**

Solutions to $I_4^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 1.00000$		
$a = 1.15372$	-5.59278	-14.0000
$b = -0.618034$		
$u = 1.00000$		
$a = -0.809017 + 0.981593I$	2.30291	-14.0000
$b = 1.61803$		
$u = 1.00000$		
$a = -0.809017 - 0.981593I$	2.30291	-14.0000
$b = 1.61803$		
$u = 1.00000$		
$a = -0.535687$	-5.59278	-14.0000
$b = -0.618034$		

## V. u-Polynomials

Crossings	u-Polynomials at each crossing
$c_1$	$(u - 2)(u^2 + u - 1)^2(u^7 - 6u^6 + 12u^5 - 11u^4 + 7u^3 - 3u^2 - 1)$ $\cdot (u^{11} - 7u^{10} + \dots - 2u + 2)$
$c_2, c_9$	$(u + 1)(u^4 - u^3 + 2u^2 - 4u + 1)(u^7 - u^6 + u^5 - 2u^4 - u^2 + 1)$ $\cdot (u^{11} + 14u^9 + 4u^8 + 46u^7 + 67u^6 - 66u^5 - 7u^4 - 30u^3 - 9u^2 - 3u - 1)$
$c_3, c_7$	$(u - 1)(u^4 - u^3 + 2u^2 - 4u + 1)(u^7 + u^6 + u^5 + 2u^4 + u^2 - 1)$ $\cdot (u^{11} + 14u^9 + 4u^8 + 46u^7 + 67u^6 - 66u^5 - 7u^4 - 30u^3 - 9u^2 - 3u - 1)$
$c_4, c_8$	$(u + 1)(u^4 + u^3 - 2u - 1)(u^7 - u^5 - 2u^3 + u^2 - u + 1)$ $\cdot (u^{11} + u^{10} + \dots - 4u - 1)$
$c_5, c_6$	$(u - 1)^5(u^7 + u^6 - 5u^5 - 5u^4 + 6u^3 + 7u^2 + u - 1)$ $\cdot (u^{11} + 5u^{10} + \dots - 10u - 4)$
$c_{10}, c_{11}$	$(u - 1)^4(u + 1)(u^7 - u^6 - 5u^5 + 5u^4 + 6u^3 - 7u^2 + u + 1)$ $\cdot (u^{11} + 5u^{10} + \dots - 10u - 4)$
$c_{12}$	$(u + 1)^5(u^7 - 5u^6 + 7u^5 - 5u^4 - 4u^3 - 7u^2 + 5u + 1)$ $\cdot (u^{11} - 3u^{10} + \dots - 3192u - 576)$

## VI. Riley Polynomials

Crossings	Riley Polynomials at each crossing
$c_1$	$(y - 4)(y^2 - 3y + 1)^2(y^7 - 12y^6 + \dots - 6y - 1)$ $\cdot (y^{11} - 31y^{10} + \dots + 200y - 4)$
$c_2, c_3, c_7$ $c_9$	$(y - 1)(y^4 + 3y^3 - 2y^2 - 12y + 1)$ $\cdot (y^7 + y^6 + \dots + 2y - 1)(y^{11} + 28y^{10} + \dots - 9y - 1)$
$c_4, c_8$	$(y - 1)(y^4 - y^3 + 2y^2 - 4y + 1)(y^7 - 2y^6 + \dots - y - 1)$ $\cdot (y^{11} + 17y^{10} + \dots + 24y - 1)$
$c_5, c_6, c_{10}$ $c_{11}$	$(y - 1)^5(y^7 - 11y^6 + 47y^5 - 97y^4 + 98y^3 - 47y^2 + 15y - 1)$ $\cdot (y^{11} - 13y^{10} + \dots + 172y - 16)$
$c_{12}$	$(y - 1)^5(y^7 - 11y^6 - 9y^5 - 141y^4 + 26y^3 - 79y^2 + 39y - 1)$ $\cdot (y^{11} + 67y^{10} + \dots + 7508160y - 331776)$