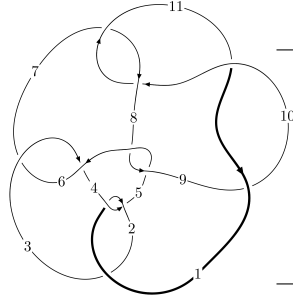
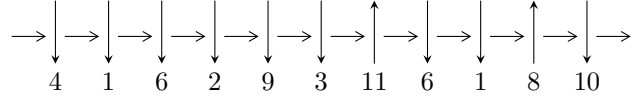


# 11n<sub>31</sub> (K11n<sub>31</sub>)



A knot diagram<sup>1</sup>

## Linearized knot diagram



## Solving Sequence

$$7, 11 \xrightarrow{c_7} 3, 8 \xrightarrow{c_6} 6 \xrightarrow{c_3} 4 \xrightarrow{c_{10}} 10 \xrightarrow{c_{11}} 1 \xrightarrow{c_2} 2 \xrightarrow{c_4} 5 \xrightarrow{c_9} 9 \twoheadrightarrow c_1, c_5, c_8$$

## Ideals for irreducible components<sup>2</sup> of $X_{\text{par}}$

$$I_1^u = \langle 7u^{10} - 8u^9 - 23u^8 + 57u^7 - 2u^6 + 8u^5 - 87u^4 + 232u^3 + 3u^2 + 164b + 47u - 106, \\ 38u^{10} - 184u^9 + 414u^8 - 493u^7 + 446u^6 - 513u^5 + 828u^4 - 650u^3 + 397u^2 + 82a - 395u + 350, \\ u^{11} - 5u^{10} + 12u^9 - 16u^8 + 16u^7 - 17u^6 + 25u^5 - 21u^4 + 14u^3 - 10u^2 + 10u - 1 \rangle$$

$$I_2^u = \langle -a^2u - 2au + b - a + u, a^3 - a^2u + 2a^2 - au - a + u - 2, u^2 + u + 1 \rangle$$

$$I_3^u = \langle b, -u^3 + 2u^2 + a - 2u, u^4 - u^3 + u^2 + 1 \rangle$$

\* 3 irreducible components of  $\dim_{\mathbb{C}} = 0$ , with total 21 representations.

<sup>1</sup>The image of knot diagram is generated by the software “**Draw programme**” developed by Andrew Bartholomew (<http://www.layer8.co.uk/maths/draw/index.htm#Running-draw>), where we modified some parts for our purpose (<https://github.com/CATsTAILs/LinksPainter>).

<sup>2</sup>All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

$$\text{I. } I_1^u = \langle 7u^{10} - 8u^9 + \dots + 164b - 106, 38u^{10} - 184u^9 + \dots + 82a + 350, u^{11} - 5u^{10} + \dots + 10u - 1 \rangle$$

(i) Arc colorings

$$a_7 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_3 = \begin{pmatrix} -0.463415u^{10} + 2.24390u^9 + \dots + 4.81707u - 4.26829 \\ -0.0426829u^{10} + 0.0487805u^9 + \dots - 0.286585u + 0.646341 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} 1 \\ -u^2 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} 0.359756u^{10} - 1.76829u^9 + \dots - 3.79878u + 3.69512 \\ 0.0121951u^{10} + 0.164634u^9 + \dots + 1.18902u - 0.506098 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} -0.871951u^{10} + 3.85366u^9 + \dots + 6.35976u - 6.43902 \\ 0.884146u^{10} - 3.68902u^9 + \dots - 5.67073u + 1.43293 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} -u \\ u^3 + u \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -u^3 \\ u^5 + u^3 + u \end{pmatrix}$$

$$a_2 = \begin{pmatrix} -0.152439u^{10} + 1.31707u^9 + \dots + 2.76220u - 4.04878 \\ -1.23780u^{10} + 5.41463u^9 + \dots + 8.68902u - 0.256098 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -0.201220u^{10} + 1.15854u^9 + \dots + 2.00610u - 3.52439 \\ -0.195122u^{10} + 3.11585u^9 + \dots + 5.22561u - 0.152439 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -u^5 - u \\ u^7 + u^5 + 2u^3 + u \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -u^5 - u \\ u^7 + u^5 + 2u^3 + u \end{pmatrix}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes

$$= -\frac{69}{82}u^{10} + \frac{319}{82}u^9 - \frac{364}{41}u^8 + \frac{460}{41}u^7 - \frac{935}{82}u^6 + \frac{476}{41}u^5 - \frac{687}{41}u^4 + \frac{520}{41}u^3 - \frac{539}{41}u^2 + \frac{673}{82}u - \frac{444}{41}$$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1, c_4$	$u^{11} - 7u^{10} + \dots - 9u + 1$
$c_2$	$u^{11} + 11u^{10} + \dots + 5u + 1$
$c_3, c_6$	$u^{11} - 6u^{10} + \dots - 24u + 16$
$c_5, c_8$	$u^{11} - 2u^{10} + \dots + 96u + 64$
$c_7, c_{10}$	$u^{11} + 5u^{10} + \dots + 10u + 1$
$c_9, c_{11}$	$u^{11} - u^{10} + \dots + 80u - 1$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1, c_4$	$y^{11} - 11y^{10} + \dots + 5y - 1$
$c_2$	$y^{11} + 113y^{10} + \dots - 3895y - 1$
$c_3, c_6$	$y^{11} + 30y^{10} + \dots - 2240y - 256$
$c_5, c_8$	$y^{11} + 52y^{10} + \dots + 33792y - 4096$
$c_7, c_{10}$	$y^{11} - y^{10} + \dots + 80y - 1$
$c_9, c_{11}$	$y^{11} + 31y^{10} + \dots + 6676y - 1$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.685415 + 0.773477I$		
$a = -0.73400 + 2.17133I$	$-1.32886 - 1.52951I$	$-7.26885 + 4.94950I$
$b = -0.460151 - 0.697009I$		
$u = -0.685415 - 0.773477I$		
$a = -0.73400 - 2.17133I$	$-1.32886 + 1.52951I$	$-7.26885 - 4.94950I$
$b = -0.460151 + 0.697009I$		
$u = 0.855917 + 0.653801I$		
$a = 0.594255 + 0.858681I$	$4.33457 + 4.30583I$	$-3.61862 - 3.76799I$
$b = 0.710923 - 1.191110I$		
$u = 0.855917 - 0.653801I$		
$a = 0.594255 - 0.858681I$	$4.33457 - 4.30583I$	$-3.61862 + 3.76799I$
$b = 0.710923 + 1.191110I$		
$u = -0.315247 + 0.806810I$		
$a = -0.545836 + 0.331424I$	$-0.33628 - 1.50726I$	$-2.98443 + 4.38710I$
$b = -0.143998 + 0.360224I$		
$u = -0.315247 - 0.806810I$		
$a = -0.545836 - 0.331424I$	$-0.33628 + 1.50726I$	$-2.98443 - 4.38710I$
$b = -0.143998 - 0.360224I$		
$u = 0.94424 + 1.31822I$		
$a = -1.30445 - 1.00484I$	$-18.0782 + 10.5314I$	$-5.95863 - 4.05407I$
$b = -1.48748 + 2.15268I$		
$u = 0.94424 - 1.31822I$		
$a = -1.30445 + 1.00484I$	$-18.0782 - 10.5314I$	$-5.95863 + 4.05407I$
$b = -1.48748 - 2.15268I$		
$u = 0.110617$		
$a = -3.78537$	$-1.00288$	$-10.0670$
$b = 0.612580$		
$u = 1.64519 + 0.99588I$		
$a = 0.882711 + 0.513382I$	$-16.1660 - 1.6365I$	$-5.13576 + 0.13357I$
$b = -1.92558 - 3.89584I$		

	Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u =$	$1.64519 - 0.99588I$		
$a =$	$0.882711 - 0.513382I$	$-16.1660 + 1.6365I$	$-5.13576 - 0.13357I$
$b =$	$-1.92558 + 3.89584I$		

$$\text{II. } I_2^u = \langle -a^2u - 2au + b - a + u, a^3 - a^2u + 2a^2 - au - a + u - 2, u^2 + u + 1 \rangle$$

(i) Arc colorings

$$a_7 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_3 = \begin{pmatrix} a \\ a^2u + 2au + a - u \end{pmatrix}$$

$$a_8 = \begin{pmatrix} 1 \\ u + 1 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} a^2u + au + a - 3u \\ -a^2 + au + 3 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} -a^2u - 2a^2 - au - 2a + 2 \\ -a^2u - a^2 - au - a + u + 2 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} -u \\ u + 1 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} a^2u + 2au + 2a - u \\ a^2u + 2au + a - u \end{pmatrix}$$

$$a_5 = \begin{pmatrix} a^2u + au + a - 3u \\ -a^2 + au + 3 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} 1 \\ u + 1 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} 1 \\ u + 1 \end{pmatrix}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes =  $-4a^2 - 2au - a - 1$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1$	$(u^3 + u^2 - 1)^2$
$c_2, c_6$	$(u^3 + u^2 + 2u + 1)^2$
$c_3$	$(u^3 - u^2 + 2u - 1)^2$
$c_4$	$(u^3 - u^2 + 1)^2$
$c_5, c_8$	$u^6$
$c_7, c_{11}$	$(u^2 + u + 1)^3$
$c_9, c_{10}$	$(u^2 - u + 1)^3$



(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1, c_4$	$(y^3 - y^2 + 2y - 1)^2$
$c_2, c_3, c_6$	$(y^3 + 3y^2 + 2y - 1)^2$
$c_5, c_8$	$y^6$
$c_7, c_9, c_{10}$ $c_{11}$	$(y^2 + y + 1)^3$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_2^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.500000 + 0.866025I$ $a = 0.901916 + 0.094973I$ $b = -0.215080 + 1.307140I$	$3.02413 + 0.79824I$	$-4.05323 - 2.24743I$
$u = -0.500000 + 0.866025I$ $a = -1.362120 + 0.277556I$ $b = -0.215080 - 1.307140I$	$3.02413 - 4.85801I$	$-7.63258 + 5.38377I$
$u = -0.500000 + 0.866025I$ $a = -2.03980 + 0.49350I$ $b = -0.569840$	$-1.11345 - 2.02988I$	$-15.8142 + 11.5861I$
$u = -0.500000 - 0.866025I$ $a = 0.901916 - 0.094973I$ $b = -0.215080 - 1.307140I$	$3.02413 - 0.79824I$	$-4.05323 + 2.24743I$
$u = -0.500000 - 0.866025I$ $a = -1.362120 - 0.277556I$ $b = -0.215080 + 1.307140I$	$3.02413 + 4.85801I$	$-7.63258 - 5.38377I$
$u = -0.500000 - 0.866025I$ $a = -2.03980 - 0.49350I$ $b = -0.569840$	$-1.11345 + 2.02988I$	$-15.8142 - 11.5861I$

$$\text{III. } I_3^u = \langle b, -u^3 + 2u^2 + a - 2u, u^4 - u^3 + u^2 + 1 \rangle$$

(i) Arc colorings

$$a_7 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_3 = \begin{pmatrix} u^3 - 2u^2 + 2u \\ 0 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} 1 \\ -u^2 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} u^3 - 2u^2 + 2u \\ 0 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} -u \\ u^3 + u \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -u^3 \\ u^3 - u^2 - 1 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} -2u^2 + 2u \\ u^3 - u^2 - 1 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} u^3 \\ -u^3 + u^2 + 1 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} u^2 + 1 \\ -u^2 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} u^2 + 1 \\ -u^2 \end{pmatrix}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes =  $-3u^3 - 3u^2 + 8u - 12$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1$	$(u - 1)^4$
$c_2, c_4$	$(u + 1)^4$
$c_3, c_6$	$u^4$
$c_5, c_9$	$u^4 - u^3 + 3u^2 - 2u + 1$
$c_7$	$u^4 - u^3 + u^2 + 1$
$c_8, c_{11}$	$u^4 + u^3 + 3u^2 + 2u + 1$
$c_{10}$	$u^4 + u^3 + u^2 + 1$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1, c_2, c_4$	$(y - 1)^4$
$c_3, c_6$	$y^4$
$c_5, c_8, c_9$ $c_{11}$	$y^4 + 5y^3 + 7y^2 + 2y + 1$
$c_7, c_{10}$	$y^4 + y^3 + 3y^2 + 2y + 1$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_3^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.351808 + 0.720342I$ $a = 0.59074 + 2.34806I$ $b = 0$	$-1.85594 - 1.41510I$	$-15.1414 + 7.6022I$
$u = -0.351808 - 0.720342I$ $a = 0.59074 - 2.34806I$ $b = 0$	$-1.85594 + 1.41510I$	$-15.1414 - 7.6022I$
$u = 0.851808 + 0.911292I$ $a = 0.409261 - 0.055548I$ $b = 0$	$5.14581 + 3.16396I$	$-0.358581 - 1.047693I$
$u = 0.851808 - 0.911292I$ $a = 0.409261 + 0.055548I$ $b = 0$	$5.14581 - 3.16396I$	$-0.358581 + 1.047693I$

#### IV. u-Polynomials

Crossings	u-Polynomials at each crossing
$c_1$	$((u-1)^4)(u^3+u^2-1)^2(u^{11}-7u^{10}+\dots-9u+1)$
$c_2$	$((u+1)^4)(u^3+u^2+2u+1)^2(u^{11}+11u^{10}+\dots+5u+1)$
$c_3$	$u^4(u^3-u^2+2u-1)^2(u^{11}-6u^{10}+\dots-24u+16)$
$c_4$	$((u+1)^4)(u^3-u^2+1)^2(u^{11}-7u^{10}+\dots-9u+1)$
$c_5$	$u^6(u^4-u^3+3u^2-2u+1)(u^{11}-2u^{10}+\dots+96u+64)$
$c_6$	$u^4(u^3+u^2+2u+1)^2(u^{11}-6u^{10}+\dots-24u+16)$
$c_7$	$((u^2+u+1)^3)(u^4-u^3+u^2+1)(u^{11}+5u^{10}+\dots+10u+1)$
$c_8$	$u^6(u^4+u^3+3u^2+2u+1)(u^{11}-2u^{10}+\dots+96u+64)$
$c_9$	$((u^2-u+1)^3)(u^4-u^3+3u^2-2u+1)(u^{11}-u^{10}+\dots+80u-1)$
$c_{10}$	$((u^2-u+1)^3)(u^4+u^3+u^2+1)(u^{11}+5u^{10}+\dots+10u+1)$
$c_{11}$	$((u^2+u+1)^3)(u^4+u^3+3u^2+2u+1)(u^{11}-u^{10}+\dots+80u-1)$

### V. Riley Polynomials

Crossings	Riley Polynomials at each crossing
$c_1, c_4$	$((y-1)^4)(y^3 - y^2 + 2y - 1)^2(y^{11} - 11y^{10} + \dots + 5y - 1)$
$c_2$	$((y-1)^4)(y^3 + 3y^2 + 2y - 1)^2(y^{11} + 113y^{10} + \dots - 3895y - 1)$
$c_3, c_6$	$y^4(y^3 + 3y^2 + 2y - 1)^2(y^{11} + 30y^{10} + \dots - 2240y - 256)$
$c_5, c_8$	$y^6(y^4 + 5y^3 + \dots + 2y + 1)(y^{11} + 52y^{10} + \dots + 33792y - 4096)$
$c_7, c_{10}$	$((y^2 + y + 1)^3)(y^4 + y^3 + 3y^2 + 2y + 1)(y^{11} - y^{10} + \dots + 80y - 1)$
$c_9, c_{11}$	$((y^2 + y + 1)^3)(y^4 + 5y^3 + \dots + 2y + 1)(y^{11} + 31y^{10} + \dots + 6676y - 1)$