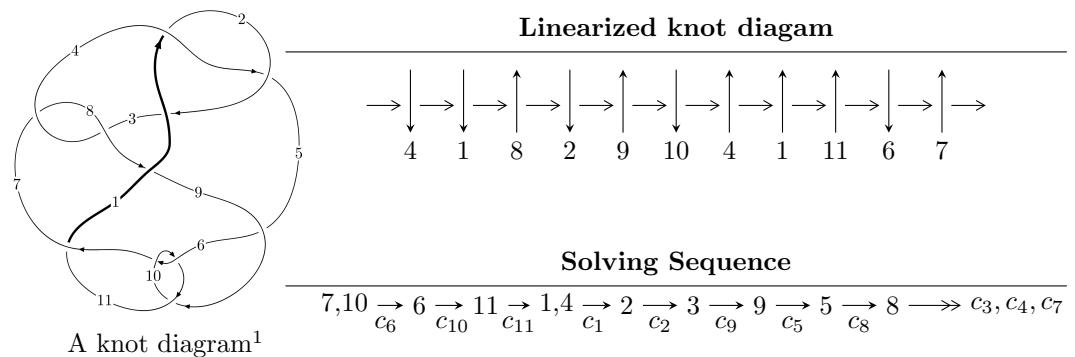


$$\frac{1}{11}n_{53} \left(K \frac{1}{11}n_{53} \right)$$



Ideals for irreducible components² of X_{par}

$$I_1^u = \langle -u^{22} + 2u^{21} + \cdots + b + 1, u^{21} - u^{20} + \cdots + 2u^3 + a, u^{23} - 2u^{22} + \cdots - 2u + 1 \rangle$$

$$I_2^u = \langle b, -u^2 + a - u - 1, u^5 + u^4 + 2u^3 + u^2 + u + 1 \rangle$$

* 2 irreducible components of $\dim_{\mathbb{C}} = 0$, with total 28 representations.

¹The image of knot diagram is generated by the software “**Draw programme**” developed by Andrew Bartholomew(<http://www.layer8.co.uk/math/draw/index.htm#Running-draw>), where we modified some parts for our purpose(<https://github.com/CATsTAILS/LinksPainter>).

²All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

$$I_1^u = \langle -u^{22} + 2u^{21} + \cdots + b + 1, u^{21} - u^{20} + \cdots + 2u^3 + a, u^{23} - 2u^{22} + \cdots - 2u + 1 \rangle$$

(i) **Arc colorings**

$$\begin{aligned} a_7 &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ a_{10} &= \begin{pmatrix} 0 \\ u \end{pmatrix} \\ a_6 &= \begin{pmatrix} 1 \\ -u^2 \end{pmatrix} \\ a_{11} &= \begin{pmatrix} -u \\ u^3 + u \end{pmatrix} \\ a_1 &= \begin{pmatrix} u^3 \\ u^3 + u \end{pmatrix} \\ a_4 &= \begin{pmatrix} -u^{21} + u^{20} + \cdots - u^4 - 2u^3 \\ u^{22} - 2u^{21} + \cdots + 2u - 1 \end{pmatrix} \\ a_2 &= \begin{pmatrix} u^{18} - u^{17} + \cdots - u^2 + u \\ -u^8 - 2u^6 - 2u^4 \end{pmatrix} \\ a_3 &= \begin{pmatrix} -u^{21} + u^{20} + \cdots + 2u - 1 \\ u^{22} - 2u^{21} + \cdots + u - 1 \end{pmatrix} \\ a_9 &= \begin{pmatrix} -u^3 \\ u^5 + u^3 + u \end{pmatrix} \\ a_5 &= \begin{pmatrix} -u^6 - u^4 + 1 \\ u^8 + 2u^6 + 2u^4 \end{pmatrix} \\ a_8 &= \begin{pmatrix} -u^{11} - 2u^9 - 2u^7 - u^3 \\ -u^{11} - 3u^9 - 4u^7 - u^5 + u^3 + u \end{pmatrix} \\ a_8 &= \begin{pmatrix} -u^{11} - 2u^9 - 2u^7 - u^3 \\ -u^{11} - 3u^9 - 4u^7 - u^5 + u^3 + u \end{pmatrix} \end{aligned}$$

(ii) **Obstruction class** = -1

(iii) **Cusp Shapes**

$$= 4u^{22} - 9u^{21} + 30u^{20} - 48u^{19} + 96u^{18} - 131u^{17} + 190u^{16} - 224u^{15} + 249u^{14} - 262u^{13} + 237u^{12} - 210u^{11} + 148u^{10} - 93u^9 + 53u^8 - 6u^7 - 14u^6 + 25u^5 - 18u^4 - 10u^2 + 9u - 5$$

(iv) u-Polynomials at the component

| Crossings | u-Polynomials at each crossing |
|---------------|--|
| c_1, c_4 | $u^{23} - 6u^{22} + \cdots + 4u - 1$ |
| c_2 | $u^{23} + 32u^{22} + \cdots - 16u + 1$ |
| c_3, c_7 | $u^{23} - u^{22} + \cdots + 32u - 32$ |
| c_5, c_{11} | $u^{23} - 2u^{22} + \cdots + 30u - 9$ |
| c_6, c_{10} | $u^{23} + 2u^{22} + \cdots - 2u - 1$ |
| c_8 | $u^{23} + 24u^{21} + \cdots + 2u - 1$ |
| c_9 | $u^{23} - 12u^{22} + \cdots - 2u + 1$ |

(v) Riley Polynomials at the component

| Crossings | Riley Polynomials at each crossing |
|---------------|---|
| c_1, c_4 | $y^{23} - 32y^{22} + \cdots - 16y - 1$ |
| c_2 | $y^{23} - 76y^{22} + \cdots + 772y - 1$ |
| c_3, c_7 | $y^{23} + 33y^{22} + \cdots + 3584y - 1024$ |
| c_5, c_{11} | $y^{23} - 12y^{22} + \cdots - 162y - 81$ |
| c_6, c_{10} | $y^{23} + 12y^{22} + \cdots - 2y - 1$ |
| c_8 | $y^{23} + 48y^{22} + \cdots - 2y - 1$ |
| c_9 | $y^{23} + 24y^{21} + \cdots - 6y - 1$ |

(vi) Complex Volumes and Cusp Shapes

| Solutions to I_1^u | $\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$ | Cusp shape |
|-----------------------------|---------------------------------------|-----------------------|
| $u = -0.700458 + 0.794163I$ | | |
| $a = -1.45877 + 0.07887I$ | $-14.1192 + 2.6463I$ | $-3.65714 - 2.84707I$ |
| $b = -0.09416 - 2.13003I$ | | |
| $u = -0.700458 - 0.794163I$ | | |
| $a = -1.45877 - 0.07887I$ | $-14.1192 - 2.6463I$ | $-3.65714 + 2.84707I$ |
| $b = -0.09416 + 2.13003I$ | | |
| $u = 0.855800 + 0.265895I$ | | |
| $a = -0.795468 - 0.188526I$ | $-11.07970 + 5.13429I$ | $-2.96602 - 2.08249I$ |
| $b = -0.45428 - 1.93201I$ | | |
| $u = 0.855800 - 0.265895I$ | | |
| $a = -0.795468 + 0.188526I$ | $-11.07970 - 5.13429I$ | $-2.96602 + 2.08249I$ |
| $b = -0.45428 + 1.93201I$ | | |
| $u = 0.366516 + 1.072630I$ | | |
| $a = 0.312367 - 0.791611I$ | $1.62847 - 1.16584I$ | $3.47504 + 0.42481I$ |
| $b = -0.215406 - 0.997383I$ | | |
| $u = 0.366516 - 1.072630I$ | | |
| $a = 0.312367 + 0.791611I$ | $1.62847 + 1.16584I$ | $3.47504 - 0.42481I$ |
| $b = -0.215406 + 0.997383I$ | | |
| $u = -0.477094 + 1.041350I$ | | |
| $a = 2.43046 - 1.25941I$ | $-0.82985 + 3.19017I$ | $0.92409 - 4.72756I$ |
| $b = 1.105500 + 0.395247I$ | | |
| $u = -0.477094 - 1.041350I$ | | |
| $a = 2.43046 + 1.25941I$ | $-0.82985 - 3.19017I$ | $0.92409 + 4.72756I$ |
| $b = 1.105500 - 0.395247I$ | | |
| $u = 0.270789 + 0.755843I$ | | |
| $a = 0.693663 + 0.260910I$ | $0.390924 - 1.217980I$ | $4.26810 + 5.45735I$ |
| $b = 0.349805 - 0.349927I$ | | |
| $u = 0.270789 - 0.755843I$ | | |
| $a = 0.693663 - 0.260910I$ | $0.390924 + 1.217980I$ | $4.26810 - 5.45735I$ |
| $b = 0.349805 + 0.349927I$ | | |

| Solutions to I_1^u | $\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$ | Cusp shape |
|-----------------------------|---------------------------------------|-----------------------|
| $u = 0.514060 + 1.104640I$ | | |
| $a = -1.31154 + 0.63853I$ | $0.55803 - 6.09572I$ | $0.93537 + 6.19372I$ |
| $b = 0.143374 + 1.329070I$ | | |
| $u = 0.514060 - 1.104640I$ | | |
| $a = -1.31154 - 0.63853I$ | $0.55803 + 6.09572I$ | $0.93537 - 6.19372I$ |
| $b = 0.143374 - 1.329070I$ | | |
| $u = -0.455035 + 1.182370I$ | | |
| $a = -1.061190 + 0.524858I$ | $5.15559 + 4.27437I$ | $9.78577 - 3.28837I$ |
| $b = -0.608111 - 0.025252I$ | | |
| $u = -0.455035 - 1.182370I$ | | |
| $a = -1.061190 - 0.524858I$ | $5.15559 - 4.27437I$ | $9.78577 + 3.28837I$ |
| $b = -0.608111 + 0.025252I$ | | |
| $u = 0.269522 + 1.238660I$ | | |
| $a = 0.24803 + 2.58133I$ | $-6.23110 + 1.56382I$ | $1.72734 - 0.04821I$ |
| $b = 0.34057 + 1.76704I$ | | |
| $u = 0.269522 - 1.238660I$ | | |
| $a = 0.24803 - 2.58133I$ | $-6.23110 - 1.56382I$ | $1.72734 + 0.04821I$ |
| $b = 0.34057 - 1.76704I$ | | |
| $u = -0.719779$ | | |
| $a = 0.526584$ | 1.81820 | 6.35980 |
| $b = 0.562756$ | | |
| $u = 0.573488 + 1.178530I$ | | |
| $a = 2.84278 - 1.00616I$ | $-8.35091 - 10.39480I$ | $-0.03516 + 5.66347I$ |
| $b = 0.57374 - 1.89861I$ | | |
| $u = 0.573488 - 1.178530I$ | | |
| $a = 2.84278 + 1.00616I$ | $-8.35091 + 10.39480I$ | $-0.03516 - 5.66347I$ |
| $b = 0.57374 + 1.89861I$ | | |
| $u = -0.469209 + 0.502755I$ | | |
| $a = -0.699918 - 0.973698I$ | $-2.46377 + 0.78545I$ | $-4.30142 - 0.09221I$ |
| $b = -0.674614 + 0.656598I$ | | |

| Solutions to I_1^u | $\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$ | Cusp shape |
|-----------------------------|---------------------------------------|-----------------------|
| $u = -0.469209 - 0.502755I$ | | |
| $a = -0.699918 + 0.973698I$ | $-2.46377 - 0.78545I$ | $-4.30142 + 0.09221I$ |
| $b = -0.674614 - 0.656598I$ | | |
| $u = 0.611511 + 0.287188I$ | | |
| $a = 0.036277 - 0.858010I$ | $-1.75612 + 1.64275I$ | $-3.33585 - 2.40342I$ |
| $b = -0.247796 + 1.071450I$ | | |
| $u = 0.611511 - 0.287188I$ | | |
| $a = 0.036277 + 0.858010I$ | $-1.75612 - 1.64275I$ | $-3.33585 + 2.40342I$ |
| $b = -0.247796 - 1.071450I$ | | |

$$\text{II. } I_2^u = \langle b, -u^2 + a - u - 1, u^5 + u^4 + 2u^3 + u^2 + u + 1 \rangle$$

(i) Arc colorings

$$a_7 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_6 = \begin{pmatrix} 1 \\ -u^2 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} -u \\ u^3 + u \end{pmatrix}$$

$$a_1 = \begin{pmatrix} u^3 \\ u^3 + u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} u^2 + u + 1 \\ 0 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} u^3 + u^2 + u + 1 \\ u^3 + u \end{pmatrix}$$

$$a_3 = \begin{pmatrix} u^2 + u + 1 \\ 0 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -u^3 \\ -u^4 - u^3 - u^2 - 1 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -u^3 \\ -u^3 - u \end{pmatrix}$$

$$a_8 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes = $-2u^4 + u^3 + 2u$

(iv) u-Polynomials at the component

| Crossings | u-Polynomials at each crossing |
|------------|-----------------------------------|
| c_1 | $(u - 1)^5$ |
| c_2, c_4 | $(u + 1)^5$ |
| c_3, c_7 | u^5 |
| c_5, c_8 | $u^5 - u^4 - 2u^3 + u^2 + u + 1$ |
| c_6 | $u^5 + u^4 + 2u^3 + u^2 + u + 1$ |
| c_9 | $u^5 + 3u^4 + 4u^3 + u^2 - u - 1$ |
| c_{10} | $u^5 - u^4 + 2u^3 - u^2 + u - 1$ |
| c_{11} | $u^5 + u^4 - 2u^3 - u^2 + u - 1$ |

(v) Riley Polynomials at the component

| Crossings | Riley Polynomials at each crossing |
|--------------------|------------------------------------|
| c_1, c_2, c_4 | $(y - 1)^5$ |
| c_3, c_7 | y^5 |
| c_5, c_8, c_{11} | $y^5 - 5y^4 + 8y^3 - 3y^2 - y - 1$ |
| c_6, c_{10} | $y^5 + 3y^4 + 4y^3 + y^2 - y - 1$ |
| c_9 | $y^5 - y^4 + 8y^3 - 3y^2 + 3y - 1$ |

(vi) Complex Volumes and Cusp Shapes

| Solutions to I_2^u | $\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$ | Cusp shape |
|-----------------------------|---------------------------------------|----------------------|
| $u = 0.339110 + 0.822375I$ | | |
| $a = 0.77780 + 1.38013I$ | $-1.31583 - 1.53058I$ | $0.02124 + 2.62456I$ |
| $b = 0$ | | |
| $u = 0.339110 - 0.822375I$ | | |
| $a = 0.77780 - 1.38013I$ | $-1.31583 + 1.53058I$ | $0.02124 - 2.62456I$ |
| $b = 0$ | | |
| $u = -0.766826$ | | |
| $a = 0.821196$ | 0.756147 | -2.67610 |
| $b = 0$ | | |
| $u = -0.455697 + 1.200150I$ | | |
| $a = -0.688402 + 0.106340I$ | $4.22763 + 4.40083I$ | $0.31681 - 3.97407I$ |
| $b = 0$ | | |
| $u = -0.455697 - 1.200150I$ | | |
| $a = -0.688402 - 0.106340I$ | $4.22763 - 4.40083I$ | $0.31681 + 3.97407I$ |
| $b = 0$ | | |

III. u-Polynomials

| Crossings | u-Polynomials at each crossing |
|------------|--|
| c_1 | $((u - 1)^5)(u^{23} - 6u^{22} + \cdots + 4u - 1)$ |
| c_2 | $((u + 1)^5)(u^{23} + 32u^{22} + \cdots - 16u + 1)$ |
| c_3, c_7 | $u^5(u^{23} - u^{22} + \cdots + 32u - 32)$ |
| c_4 | $((u + 1)^5)(u^{23} - 6u^{22} + \cdots + 4u - 1)$ |
| c_5 | $(u^5 - u^4 - 2u^3 + u^2 + u + 1)(u^{23} - 2u^{22} + \cdots + 30u - 9)$ |
| c_6 | $(u^5 + u^4 + 2u^3 + u^2 + u + 1)(u^{23} + 2u^{22} + \cdots - 2u - 1)$ |
| c_8 | $(u^5 - u^4 - 2u^3 + u^2 + u + 1)(u^{23} + 24u^{21} + \cdots + 2u - 1)$ |
| c_9 | $(u^5 + 3u^4 + 4u^3 + u^2 - u - 1)(u^{23} - 12u^{22} + \cdots - 2u + 1)$ |
| c_{10} | $(u^5 - u^4 + 2u^3 - u^2 + u - 1)(u^{23} + 2u^{22} + \cdots - 2u - 1)$ |
| c_{11} | $(u^5 + u^4 - 2u^3 - u^2 + u - 1)(u^{23} - 2u^{22} + \cdots + 30u - 9)$ |

IV. Riley Polynomials

| Crossings | Riley Polynomials at each crossing |
|---------------|--|
| c_1, c_4 | $((y - 1)^5)(y^{23} - 32y^{22} + \cdots - 16y - 1)$ |
| c_2 | $((y - 1)^5)(y^{23} - 76y^{22} + \cdots + 772y - 1)$ |
| c_3, c_7 | $y^5(y^{23} + 33y^{22} + \cdots + 3584y - 1024)$ |
| c_5, c_{11} | $(y^5 - 5y^4 + 8y^3 - 3y^2 - y - 1)(y^{23} - 12y^{22} + \cdots - 162y - 81)$ |
| c_6, c_{10} | $(y^5 + 3y^4 + 4y^3 + y^2 - y - 1)(y^{23} + 12y^{22} + \cdots - 2y - 1)$ |
| c_8 | $(y^5 - 5y^4 + 8y^3 - 3y^2 - y - 1)(y^{23} + 48y^{22} + \cdots - 2y - 1)$ |
| c_9 | $(y^5 - y^4 + 8y^3 - 3y^2 + 3y - 1)(y^{23} + 24y^{22} + \cdots - 6y - 1)$ |