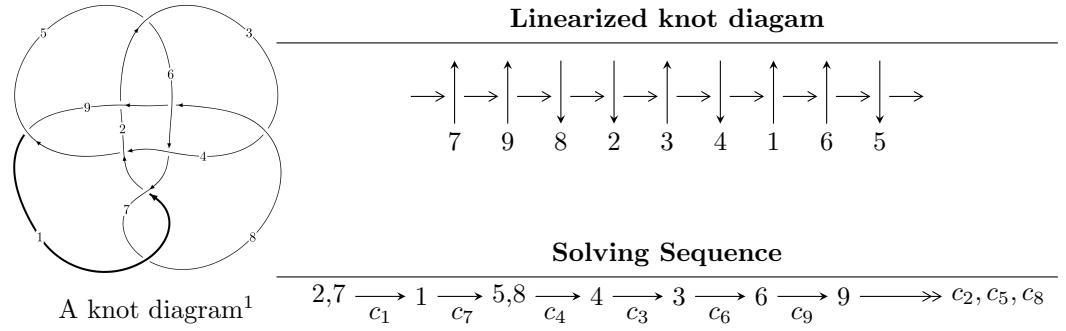


9<sub>34</sub> (K9a<sub>28</sub>)



**Ideals for irreducible components<sup>2</sup> of  $X_{\text{par}}$**

$$I_1^u = \langle -61u^{16} - 298u^{15} + \dots + 43b + 351, -107u^{16} - 319u^{15} + \dots + 172a - 257, \\ u^{17} + 5u^{16} + \dots - 21u - 4 \rangle$$

$$I_2^u = \langle u^{10}a - 2u^9a + \dots + a - 1, -u^9a - u^{10} + \dots + a^2 + 1, \\ u^{11} - 3u^{10} + 8u^9 - 13u^8 + 18u^7 - 20u^6 + 18u^5 - 15u^4 + 9u^3 - 5u^2 + 2u - 1 \rangle$$

$$I_3^u = \langle -u^3 + 2u^2 + b - 2u + 1, u^4 - u^3 + a + u - 2, u^5 - 2u^4 + 3u^3 - 3u^2 + u - 1 \rangle$$

\* 3 irreducible components of  $\dim_{\mathbb{C}} = 0$ , with total 44 representations.

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<sup>1</sup>The image of knot diagram is generated by the software “**Draw programme**” developed by Andrew Bartholomew(<http://www.layer8.co.uk/math/draw/index.htm#Running-draw>), where we modified some parts for our purpose(<https://github.com/CATsTAILs/LinksPainter>).

<sup>2</sup>All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

$$\text{I. } I_1^u = \langle -61u^{16} - 298u^{15} + \cdots + 43b + 351, -107u^{16} - 319u^{15} + \cdots + 172a - 257, u^{17} + 5u^{16} + \cdots - 21u - 4 \rangle$$

(i) **Arc colorings**

$$a_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_1 = \begin{pmatrix} 1 \\ u^2 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} 0.622093u^{16} + 1.85465u^{15} + \cdots + 5.02907u + 1.49419 \\ 1.41860u^{16} + 6.93023u^{15} + \cdots - 36.1860u - 8.16279 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} u \\ u^3 + u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 2.04070u^{16} + 8.78488u^{15} + \cdots - 31.1570u - 6.66860 \\ 1.41860u^{16} + 6.93023u^{15} + \cdots - 36.1860u - 8.16279 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} 0.901163u^{16} + 3.80814u^{15} + \cdots - 14.7616u - 2.94767 \\ 0.720930u^{16} + 3.04651u^{15} + \cdots - 9.20930u - 1.55814 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -3.56395u^{16} - 14.4477u^{15} + \cdots + 49.8895u + 13.1221 \\ -3.37209u^{16} - 15.6047u^{15} + \cdots + 62.7209u + 14.2558 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -0.843023u^{16} - 4.40116u^{15} + \cdots + 26.6802u + 7.56395 \\ -0.604651u^{16} - 2.23256u^{15} + \cdots + 18.0465u + 5.79070 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -0.843023u^{16} - 4.40116u^{15} + \cdots + 26.6802u + 7.56395 \\ -0.604651u^{16} - 2.23256u^{15} + \cdots + 18.0465u + 5.79070 \end{pmatrix}$$

(ii) **Obstruction class** = -1

(iii) **Cusp Shapes** =  $\frac{441}{43}u^{16} + \frac{1969}{43}u^{15} + \cdots - \frac{7549}{43}u - \frac{2042}{43}$

**(iv) u-Polynomials at the component**

Crossings	u-Polynomials at each crossing
$c_1, c_7$	$u^{17} + 5u^{16} + \cdots - 21u - 4$
$c_2, c_8$	$u^{17} + u^{16} + \cdots - u - 1$
$c_3, c_9$	$u^{17} + 5u^{13} + \cdots + 4u - 1$
$c_4, c_6$	$u^{17} + 2u^{16} + \cdots + 8u - 1$
$c_5$	$u^{17} + 10u^{16} + \cdots + 5u + 2$

**(v) Riley Polynomials at the component**

Crossings	Riley Polynomials at each crossing
$c_1, c_7$	$y^{17} + 9y^{16} + \cdots + 17y - 16$
$c_2, c_8$	$y^{17} + 7y^{16} + \cdots - 19y - 1$
$c_3, c_9$	$y^{17} + 10y^{15} + \cdots + 10y - 1$
$c_4, c_6$	$y^{17} - 12y^{16} + \cdots + 46y - 1$
$c_5$	$y^{17} + 16y^{15} + \cdots - 11y - 4$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.048681 + 1.008070I$		
$a = -1.66750 + 0.61626I$	$-3.35770 + 0.12402I$	$-5.97884 + 0.38118I$
$b = 1.134670 + 0.483593I$		
$u = 0.048681 - 1.008070I$		
$a = -1.66750 - 0.61626I$	$-3.35770 - 0.12402I$	$-5.97884 - 0.38118I$
$b = 1.134670 - 0.483593I$		
$u = 0.423210 + 0.769632I$		
$a = 0.879104 + 0.306597I$	$0.28619 + 1.83578I$	$2.59246 - 3.36751I$
$b = -0.311039 - 0.398365I$		
$u = 0.423210 - 0.769632I$		
$a = 0.879104 - 0.306597I$	$0.28619 - 1.83578I$	$2.59246 + 3.36751I$
$b = -0.311039 + 0.398365I$		
$u = -1.115480 + 0.170377I$		
$a = 0.027795 - 0.216323I$	$0.27750 + 8.29795I$	$1.06571 - 6.88359I$
$b = -0.973543 + 0.694225I$		
$u = -1.115480 - 0.170377I$		
$a = 0.027795 + 0.216323I$	$0.27750 - 8.29795I$	$1.06571 + 6.88359I$
$b = -0.973543 - 0.694225I$		
$u = 1.18539$		
$a = 0.285468$	2.39123	15.5890
$b = -0.154842$		
$u = -0.546851 + 1.063670I$		
$a = -0.818209 + 0.890659I$	$-3.79067 - 2.00597I$	$-6.21078 + 1.26630I$
$b = 1.249560 + 0.062335I$		
$u = -0.546851 - 1.063670I$		
$a = -0.818209 - 0.890659I$	$-3.79067 + 2.00597I$	$-6.21078 - 1.26630I$
$b = 1.249560 - 0.062335I$		
$u = -0.437546 + 1.154220I$		
$a = -1.81788 + 0.29672I$	$-4.31147 - 5.61068I$	$-7.96642 + 8.06049I$
$b = 1.40541 + 1.07727I$		

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.437546 - 1.154220I$		
$a = -1.81788 - 0.29672I$	$-4.31147 + 5.61068I$	$-7.96642 - 8.06049I$
$b = 1.40541 - 1.07727I$		
$u = -0.582313 + 0.090917I$		
$a = 0.732188 - 0.199615I$	$-1.32135 + 1.62186I$	$-2.58195 - 4.11393I$
$b = 0.842156 - 0.620975I$		
$u = -0.582313 - 0.090917I$		
$a = 0.732188 + 0.199615I$	$-1.32135 - 1.62186I$	$-2.58195 + 4.11393I$
$b = 0.842156 + 0.620975I$		
$u = -0.59542 + 1.30831I$		
$a = 1.58913 - 0.22054I$	$-3.3114 - 14.3446I$	$-1.18187 + 8.40363I$
$b = -1.40015 - 0.93567I$		
$u = -0.59542 - 1.30831I$		
$a = 1.58913 + 0.22054I$	$-3.3114 + 14.3446I$	$-1.18187 - 8.40363I$
$b = -1.40015 + 0.93567I$		
$u = -0.28698 + 1.44004I$		
$a = 0.807648 - 0.511056I$	$-5.40594 + 3.12036I$	$-5.53287 - 3.71986I$
$b = -0.869639 - 0.080492I$		
$u = -0.28698 - 1.44004I$		
$a = 0.807648 + 0.511056I$	$-5.40594 - 3.12036I$	$-5.53287 + 3.71986I$
$b = -0.869639 + 0.080492I$		

$$I_2^u = \langle u^{10}a - 2u^9a + \dots + a - 1, -u^9a - u^{10} + \dots + a^2 + 1, u^{11} - 3u^{10} + \dots + 2u - 1 \rangle$$

(i) **Arc colorings**

$$a_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_1 = \begin{pmatrix} 1 \\ u^2 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} a \\ -u^{10}a + 2u^9a + \dots - a + 1 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} u \\ u^3 + u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} -u^{10}a + 2u^9a + \dots - 2u + 1 \\ -u^{10}a + 2u^9a + \dots - a + 1 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} u^9 - 2u^8 + 4u^7 - 3u^6 + u^5 + u^3a + u^4 - 4u^3 + au + 2u^2 + a - 3u + 1 \\ u^{10}a - 2u^9a + \dots + a + 1 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -u^9a + u^9 + \dots - a + 2 \\ 1 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} u^9 - 4u^8 + \dots + a - 1 \\ -u^{10}a + 2u^9a + \dots - a + 1 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} u^9 - 4u^8 + \dots + a - 1 \\ -u^{10}a + 2u^9a + \dots - a + 1 \end{pmatrix}$$

(ii) **Obstruction class** = -1

(iii) **Cusp Shapes** =  $4u^{10} - 4u^9 + 8u^8 + 4u^7 - 8u^6 + 12u^5 - 12u^4 + 4u^3 - 8u^2 + 2$

**(iv) u-Polynomials at the component**

Crossings	u-Polynomials at each crossing
$c_1, c_7$	$(u^{11} - 3u^{10} + \cdots + 2u - 1)^2$
$c_2, c_8$	$u^{22} + 3u^{21} + \cdots + 6u + 1$
$c_3, c_9$	$u^{22} + u^{21} + \cdots - 10u + 1$
$c_4, c_6$	$u^{22} - u^{21} + \cdots - 4u + 1$
$c_5$	$(u^{11} - 5u^{10} + 12u^9 - 15u^8 + 8u^7 + 4u^6 - 8u^5 + 3u^4 + 3u^3 - 3u^2 + 1)^2$

**(v) Riley Polynomials at the component**

Crossings	Riley Polynomials at each crossing
$c_1, c_7$	$(y^{11} + 7y^{10} + \cdots - 6y - 1)^2$
$c_2, c_8$	$y^{22} - 5y^{21} + \cdots + 72y^2 + 1$
$c_3, c_9$	$y^{22} - y^{21} + \cdots - 8y + 1$
$c_4, c_6$	$y^{22} + 3y^{21} + \cdots + 8y + 1$
$c_5$	$(y^{11} - y^{10} + \cdots + 6y - 1)^2$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_2^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.253759 + 0.946686I$		
$a = -0.049055 - 1.213920I$	$-0.13765 - 5.21629I$	$0.43603 + 9.01278I$
$b = -0.33551 + 1.93421I$		
$u = -0.253759 + 0.946686I$		
$a = -2.57911 - 0.36655I$	$-0.13765 - 5.21629I$	$0.43603 + 9.01278I$
$b = 0.584301 + 0.546847I$		
$u = -0.253759 - 0.946686I$		
$a = -0.049055 + 1.213920I$	$-0.13765 + 5.21629I$	$0.43603 - 9.01278I$
$b = -0.33551 - 1.93421I$		
$u = -0.253759 - 0.946686I$		
$a = -2.57911 + 0.36655I$	$-0.13765 + 5.21629I$	$0.43603 - 9.01278I$
$b = 0.584301 - 0.546847I$		
$u = 1.10821$		
$a = 0.305204 + 0.028042I$	2.37876	12.2610
$b = -0.160435 - 0.287182I$		
$u = 1.10821$		
$a = 0.305204 - 0.028042I$	2.37876	12.2610
$b = -0.160435 + 0.287182I$		
$u = 0.572881 + 0.536287I$		
$a = 0.605018 - 0.138715I$	$0.42400 + 2.24779I$	$3.63582 - 5.06360I$
$b = 0.379406 - 0.599968I$		
$u = 0.572881 + 0.536287I$		
$a = 1.12964 + 0.99333I$	$0.42400 + 2.24779I$	$3.63582 - 5.06360I$
$b = -0.960104 - 0.104756I$		
$u = 0.572881 - 0.536287I$		
$a = 0.605018 + 0.138715I$	$0.42400 - 2.24779I$	$3.63582 + 5.06360I$
$b = 0.379406 + 0.599968I$		
$u = 0.572881 - 0.536287I$		
$a = 1.12964 - 0.99333I$	$0.42400 - 2.24779I$	$3.63582 + 5.06360I$
$b = -0.960104 + 0.104756I$		

Solutions to $I_2^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.290349 + 1.272230I$		
$a = 1.21435 + 0.88581I$	$-4.63073 + 5.00074I$	$-7.84059 - 6.22751I$
$b = -0.734695 + 0.377618I$		
$u = 0.290349 + 1.272230I$		
$a = -1.62496 + 0.36379I$	$-4.63073 + 5.00074I$	$-7.84059 - 6.22751I$
$b = 1.46811 - 0.97707I$		
$u = 0.290349 - 1.272230I$		
$a = 1.21435 - 0.88581I$	$-4.63073 - 5.00074I$	$-7.84059 + 6.22751I$
$b = -0.734695 - 0.377618I$		
$u = 0.290349 - 1.272230I$		
$a = -1.62496 - 0.36379I$	$-4.63073 - 5.00074I$	$-7.84059 + 6.22751I$
$b = 1.46811 + 0.97707I$		
$u = -0.234018 + 0.605151I$		
$a = 0.357585 - 0.648167I$	$0.80290 + 2.70441I$	$3.46762 + 0.08333I$
$b = 0.378854 - 1.068730I$		
$u = -0.234018 + 0.605151I$		
$a = 2.61356 + 0.79794I$	$0.80290 + 2.70441I$	$3.46762 + 0.08333I$
$b = -0.866642 - 0.847442I$		
$u = -0.234018 - 0.605151I$		
$a = 0.357585 + 0.648167I$	$0.80290 - 2.70441I$	$3.46762 - 0.08333I$
$b = 0.378854 + 1.068730I$		
$u = -0.234018 - 0.605151I$		
$a = 2.61356 - 0.79794I$	$0.80290 - 2.70441I$	$3.46762 - 0.08333I$
$b = -0.866642 + 0.847442I$		
$u = 0.57044 + 1.34258I$		
$a = -0.862107 + 0.035474I$	$-1.76023 + 5.92443I$	$3.17045 - 10.02355I$
$b = 0.818255 - 0.852218I$		
$u = 0.57044 + 1.34258I$		
$a = 1.389870 + 0.219943I$	$-1.76023 + 5.92443I$	$3.17045 - 10.02355I$
$b = -1.071530 + 0.524779I$		

Solutions to $I_2^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.57044 - 1.34258I$		
$a = -0.862107 - 0.035474I$	$-1.76023 - 5.92443I$	$3.17045 + 10.02355I$
$b = 0.818255 + 0.852218I$		
$u = 0.57044 - 1.34258I$		
$a = 1.389870 - 0.219943I$	$-1.76023 - 5.92443I$	$3.17045 + 10.02355I$
$b = -1.071530 - 0.524779I$		

### III.

$$I_3^u = \langle -u^3 + 2u^2 + b - 2u + 1, \ u^4 - u^3 + a + u - 2, \ u^5 - 2u^4 + 3u^3 - 3u^2 + u - 1 \rangle$$

(i) **Arc colorings**

$$a_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_1 = \begin{pmatrix} 1 \\ u^2 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -u^4 + u^3 - u + 2 \\ u^3 - 2u^2 + 2u - 1 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} u \\ u^3 + u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} -u^4 + 2u^3 - 2u^2 + u + 1 \\ u^3 - 2u^2 + 2u - 1 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} -u^4 + u^3 - u^2 + 2 \\ -u^4 + 2u^3 - 3u^2 + 2u - 1 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -u^4 + 3u^3 - 4u^2 + 4u - 2 \\ u^4 - u^3 + u^2 - 1 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} u^4 - 2u^3 + 3u^2 - 2u \\ u^2 - u + 1 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} u^4 - 2u^3 + 3u^2 - 2u \\ u^2 - u + 1 \end{pmatrix}$$

(ii) **Obstruction class = 1**

(iii) **Cusp Shapes** =  $-6u^4 + 13u^3 - 22u^2 + 14u - 9$

**(iv) u-Polynomials at the component**

Crossings	u-Polynomials at each crossing
$c_1$	$u^5 - 2u^4 + 3u^3 - 3u^2 + u - 1$
$c_2, c_8$	$u^5 - u^4 - u^3 + u^2 - 1$
$c_3, c_9$	$u^5 - u^3 + u^2 + u - 1$
$c_4, c_6$	$u^5 + 2u^4 + 3u^3 + 3u^2 + 3u + 1$
$c_5$	$u^5 - 3u^4 + 5u^3 - 4u^2 + 3u - 1$
$c_7$	$u^5 + 2u^4 + 3u^3 + 3u^2 + u + 1$

**(v) Riley Polynomials at the component**

Crossings	Riley Polynomials at each crossing
$c_1, c_7$	$y^5 + 2y^4 - y^3 - 7y^2 - 5y - 1$
$c_2, c_8$	$y^5 - 3y^4 + 3y^3 - 3y^2 + 2y - 1$
$c_3, c_9$	$y^5 - 2y^4 + 3y^3 - 3y^2 + 3y - 1$
$c_4, c_6$	$y^5 + 2y^4 + 3y^3 + 5y^2 + 3y - 1$
$c_5$	$y^5 + y^4 + 7y^3 + 8y^2 + y - 1$

**(vi) Complex Volumes and Cusp Shapes**

Solutions to $I_3^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.372466 + 1.263920I$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	
$a = -1.347300 - 0.010044I$	$-3.01018 + 5.17259I$	$-1.67537 - 5.94701I$
$b = 0.929085 - 0.848284I$		
$u = 0.372466 - 1.263920I$		
$a = -1.347300 + 0.010044I$	$-3.01018 - 5.17259I$	$-1.67537 + 5.94701I$
$b = 0.929085 + 0.848284I$		
$u = 1.33263$		
$a = -0.119827$	2.14584	-17.5700
$b = 0.480071$		
$u = -0.038780 + 0.656277I$		
$a = 1.90721 - 0.97967I$	0.29233 - 3.70382I	-0.53969 + 6.40947I
$b = -0.169121 + 1.134660I$		
$u = -0.038780 - 0.656277I$		
$a = 1.90721 + 0.97967I$	0.29233 + 3.70382I	-0.53969 - 6.40947I
$b = -0.169121 - 1.134660I$		

#### IV. u-Polynomials

Crossings	u-Polynomials at each crossing
$c_1$	$(u^5 - 2u^4 + 3u^3 - 3u^2 + u - 1)(u^{11} - 3u^{10} + \dots + 2u - 1)^2$ $\cdot (u^{17} + 5u^{16} + \dots - 21u - 4)$
$c_2, c_8$	$(u^5 - u^4 - u^3 + u^2 - 1)(u^{17} + u^{16} + \dots - u - 1)(u^{22} + 3u^{21} + \dots + 6u + 1)$
$c_3, c_9$	$(u^5 - u^3 + u^2 + u - 1)(u^{17} + 5u^{13} + \dots + 4u - 1)$ $\cdot (u^{22} + u^{21} + \dots - 10u + 1)$
$c_4, c_6$	$(u^5 + 2u^4 + 3u^3 + 3u^2 + 3u + 1)(u^{17} + 2u^{16} + \dots + 8u - 1)$ $\cdot (u^{22} - u^{21} + \dots - 4u + 1)$
$c_5$	$(u^5 - 3u^4 + 5u^3 - 4u^2 + 3u - 1)$ $\cdot (u^{11} - 5u^{10} + 12u^9 - 15u^8 + 8u^7 + 4u^6 - 8u^5 + 3u^4 + 3u^3 - 3u^2 + 1)^2$ $\cdot (u^{17} + 10u^{16} + \dots + 5u + 2)$
$c_7$	$(u^5 + 2u^4 + 3u^3 + 3u^2 + u + 1)(u^{11} - 3u^{10} + \dots + 2u - 1)^2$ $\cdot (u^{17} + 5u^{16} + \dots - 21u - 4)$

## V. Riley Polynomials

Crossings	Riley Polynomials at each crossing
$c_1, c_7$	$(y^5 + 2y^4 - y^3 - 7y^2 - 5y - 1)(y^{11} + 7y^{10} + \dots - 6y - 1)^2$ $\cdot (y^{17} + 9y^{16} + \dots + 17y - 16)$
$c_2, c_8$	$(y^5 - 3y^4 + 3y^3 - 3y^2 + 2y - 1)(y^{17} + 7y^{16} + \dots - 19y - 1)$ $\cdot (y^{22} - 5y^{21} + \dots + 72y^2 + 1)$
$c_3, c_9$	$(y^5 - 2y^4 + 3y^3 - 3y^2 + 3y - 1)(y^{17} + 10y^{15} + \dots + 10y - 1)$ $\cdot (y^{22} - y^{21} + \dots - 8y + 1)$
$c_4, c_6$	$(y^5 + 2y^4 + 3y^3 + 5y^2 + 3y - 1)(y^{17} - 12y^{16} + \dots + 46y - 1)$ $\cdot (y^{22} + 3y^{21} + \dots + 8y + 1)$
$c_5$	$(y^5 + y^4 + 7y^3 + 8y^2 + y - 1)(y^{11} - y^{10} + \dots + 6y - 1)^2$ $\cdot (y^{17} + 16y^{15} + \dots - 11y - 4)$