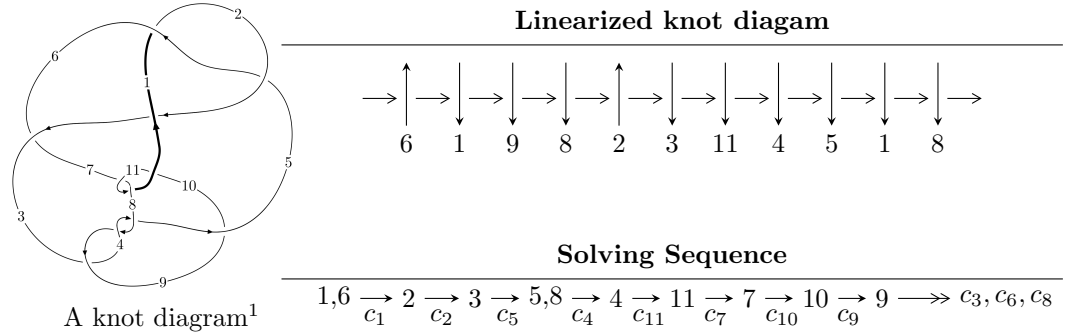


11n<sub>90</sub> (K11n<sub>90</sub>)



A knot diagram<sup>1</sup>

**Ideals for irreducible components<sup>2</sup> of  $X_{\text{par}}$**

$$I_1^u = \langle -1238113u^{25} + 2413455u^{24} + \dots + 2221939b + 1586090, \\ 4519597u^{25} - 816584u^{24} + \dots + 13331634a - 13110217, u^{26} - 2u^{25} + \dots - u + 3 \rangle$$

$$I_2^u = \langle b - 1, a^2 - 2au + 2a + u - 2, u^2 - u + 1 \rangle$$

$$I_3^u = \langle b + 1, a - u - 1, u^2 + u + 1 \rangle$$

\* 3 irreducible components of  $\dim_{\mathbb{C}} = 0$ , with total 32 representations.

<sup>1</sup>The image of knot diagram is generated by the software “**Draw programme**” developed by Andrew Bartholomew(<http://www.layer8.co.uk/maths/draw/index.htm#Running-draw>), where we modified some parts for our purpose(<https://github.com/CATsTAILs/LinksPainter>).

<sup>2</sup>All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

I.

$$I_1^u = \langle -1.24 \times 10^6 u^{25} + 2.41 \times 10^6 u^{24} + \dots + 2.22 \times 10^6 b + 1.59 \times 10^6, 4.52 \times 10^6 u^{25} - 8.17 \times 10^5 u^{24} + \dots + 1.33 \times 10^7 a - 1.31 \times 10^7, u^{26} - 2u^{25} + \dots - u + 3 \rangle$$

(i) Arc colorings

$$a_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_2 = \begin{pmatrix} 1 \\ -u^2 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} u^2 + 1 \\ -u^2 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -u \\ u^3 + u \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -0.339013u^{25} + 0.0612516u^{24} + \dots + 0.0332859u + 0.983392 \\ 0.557222u^{25} - 1.08619u^{24} + \dots + 1.41616u - 0.713831 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} -0.254668u^{25} + 0.460389u^{24} + \dots - 3.72725u - 2.65550 \\ 0.0489476u^{25} - 0.702694u^{24} + \dots + 2.91017u - 0.764005 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 0.321877u^{25} + 0.0594569u^{24} + \dots - 3.45009u + 0.683981 \\ -0.681651u^{25} + 1.22699u^{24} + \dots - 0.0147380u + 0.770149 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -u^5 - 2u^3 - u \\ u^5 + u^3 + u \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} -0.359774u^{25} + 1.28645u^{24} + \dots - 3.46483u + 1.45413 \\ -0.681651u^{25} + 1.22699u^{24} + \dots - 0.0147380u + 0.770149 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} 0.237944u^{25} + 0.0813342u^{24} + \dots - 2.32111u + 1.17821 \\ -0.616774u^{25} + 1.26410u^{24} + \dots + 0.644379u + 1.01704 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} 0.237944u^{25} + 0.0813342u^{24} + \dots - 2.32111u + 1.17821 \\ -0.616774u^{25} + 1.26410u^{24} + \dots + 0.644379u + 1.01704 \end{pmatrix}$$

(ii) Obstruction class = -1

$$(iii) \text{ Cusp Shapes} = -\frac{1151955}{2221939}u^{25} + \frac{814709}{2221939}u^{24} + \dots - \frac{5153343}{2221939}u - \frac{28973115}{2221939}$$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1, c_5$	$u^{26} - 2u^{25} + \dots - u + 3$
$c_2$	$u^{26} + 16u^{25} + \dots - 43u + 9$
$c_3, c_4, c_8$	$u^{26} + u^{25} + \dots - 8u - 4$
$c_6$	$u^{26} + 2u^{25} + \dots - 13u + 3$
$c_7, c_{11}$	$u^{26} + 3u^{25} + \dots + 22u - 3$
$c_9$	$u^{26} - u^{25} + \dots - 32u - 4$
$c_{10}$	$u^{26} + 33u^{25} + \dots + 64u + 9$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1, c_5$	$y^{26} + 16y^{25} + \dots - 43y + 9$
$c_2$	$y^{26} - 8y^{25} + \dots - 7123y + 81$
$c_3, c_4, c_8$	$y^{26} + 21y^{25} + \dots + 64y + 16$
$c_6$	$y^{26} - 32y^{25} + \dots - 187y + 9$
$c_7, c_{11}$	$y^{26} - 33y^{25} + \dots - 64y + 9$
$c_9$	$y^{26} - 39y^{25} + \dots - 128y + 16$
$c_{10}$	$y^{26} - 73y^{25} + \dots + 35108y + 81$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.987320 + 0.168214I$ $a = 1.297460 + 0.513052I$ $b = -1.63497 - 0.20181I$	$-5.22414 - 5.39338I$	$-6.45106 + 2.82273I$
$u = 0.987320 - 0.168214I$ $a = 1.297460 - 0.513052I$ $b = -1.63497 + 0.20181I$	$-5.22414 + 5.39338I$	$-6.45106 - 2.82273I$
$u = -1.01037$ $a = -1.32902$ $b = 1.68442$	$-9.37437$	$-9.45940$
$u = -0.541900 + 0.798242I$ $a = -1.229580 - 0.630085I$ $b = 0.190153 + 0.181187I$	$4.95516 - 2.19764I$	$0.54342 + 3.86213I$
$u = -0.541900 - 0.798242I$ $a = -1.229580 + 0.630085I$ $b = 0.190153 - 0.181187I$	$4.95516 + 2.19764I$	$0.54342 - 3.86213I$
$u = 0.280901 + 0.919746I$ $a = 0.066362 - 0.266060I$ $b = 0.270359 + 0.442643I$	$-0.60039 + 1.42912I$	$-6.05587 - 3.68708I$
$u = 0.280901 - 0.919746I$ $a = 0.066362 + 0.266060I$ $b = 0.270359 - 0.442643I$	$-0.60039 - 1.42912I$	$-6.05587 + 3.68708I$
$u = -0.086149 + 0.939073I$ $a = 1.12447 + 1.41361I$ $b = 1.170170 - 0.263604I$	$1.87196 - 0.46648I$	$-9.69334 - 0.39377I$
$u = -0.086149 - 0.939073I$ $a = 1.12447 - 1.41361I$ $b = 1.170170 + 0.263604I$	$1.87196 + 0.46648I$	$-9.69334 + 0.39377I$
$u = -0.714585 + 0.848170I$ $a = 1.111130 + 0.871548I$ $b = -1.336900 + 0.007719I$	$-0.16795 - 2.70526I$	$-6.45261 + 3.54399I$

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.714585 - 0.848170I$		
$a = 1.111130 - 0.871548I$	$-0.16795 + 2.70526I$	$-6.45261 - 3.54399I$
$b = -1.336900 - 0.007719I$		
$u = 0.409972 + 1.042740I$		
$a = -0.333921 + 0.008202I$	$-0.71901 + 1.35928I$	$-6.71358 - 0.21049I$
$b = 0.687191 + 0.474750I$		
$u = 0.409972 - 1.042740I$		
$a = -0.333921 - 0.008202I$	$-0.71901 - 1.35928I$	$-6.71358 + 0.21049I$
$b = 0.687191 - 0.474750I$		
$u = -0.232752 + 1.110800I$		
$a = -0.406380 + 1.143680I$	$-3.76323 - 2.26383I$	$-13.05428 + 2.02208I$
$b = -0.979109 - 0.571742I$		
$u = -0.232752 - 1.110800I$		
$a = -0.406380 - 1.143680I$	$-3.76323 + 2.26383I$	$-13.05428 - 2.02208I$
$b = -0.979109 + 0.571742I$		
$u = 0.432711 + 1.187740I$		
$a = 0.517506 + 1.268290I$	$-0.86443 + 6.41567I$	$-7.45843 - 6.37638I$
$b = 0.659883 - 0.866942I$		
$u = 0.432711 - 1.187740I$		
$a = 0.517506 - 1.268290I$	$-0.86443 - 6.41567I$	$-7.45843 + 6.37638I$
$b = 0.659883 + 0.866942I$		
$u = 0.683039 + 0.071498I$		
$a = -0.81507 - 1.50432I$	$2.38839 - 2.21658I$	$-3.00360 + 3.59199I$
$b = 0.587913 + 0.647297I$		
$u = 0.683039 - 0.071498I$		
$a = -0.81507 + 1.50432I$	$2.38839 + 2.21658I$	$-3.00360 - 3.59199I$
$b = 0.587913 - 0.647297I$		
$u = 0.576371 + 1.269310I$		
$a = -0.02214 - 1.67156I$	$-8.60064 + 11.02740I$	$-8.81919 - 5.78425I$
$b = -1.66528 + 0.31996I$		

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.576371 - 1.269310I$ $a = -0.02214 + 1.67156I$ $b = -1.66528 - 0.31996I$	$-8.60064 - 11.02740I$	$-8.81919 + 5.78425I$
$u = 0.376370 + 1.343310I$ $a = -0.157273 - 0.643532I$ $b = -1.74770 - 0.06929I$	$-10.10760 - 0.68348I$	$-10.33009 + 0.33748I$
$u = 0.376370 - 1.343310I$ $a = -0.157273 + 0.643532I$ $b = -1.74770 + 0.06929I$	$-10.10760 + 0.68348I$	$-10.33009 - 0.33748I$
$u = -0.49655 + 1.32834I$ $a = 0.015741 - 1.194450I$ $b = 1.76275 + 0.15335I$	$-13.5263 - 5.3591I$	$-12.04516 + 3.16064I$
$u = -0.49655 - 1.32834I$ $a = 0.015741 + 1.194450I$ $b = 1.76275 - 0.15335I$	$-13.5263 + 5.3591I$	$-12.04516 - 3.16064I$
$u = -0.339124$ $a = 1.65907$ $b = -0.613341$	$-0.865956$	$-11.4730$

$$\text{II. } I_2^u = \langle b - 1, a^2 - 2au + 2a + u - 2, u^2 - u + 1 \rangle$$

(i) Arc colorings

$$a_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_2 = \begin{pmatrix} 1 \\ -u + 1 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} u \\ -u + 1 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -u \\ u - 1 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} a \\ 1 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} -au + u - 1 \\ au - a + 2u - 1 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} -a + 1 \\ -1 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} -a \\ -1 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -u + 1 \\ -au - 2 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -u + 1 \\ -au - 2 \end{pmatrix}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes =  $-4u - 4$



(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1, c_6$	$(u^2 - u + 1)^2$
$c_2, c_5$	$(u^2 + u + 1)^2$
$c_3, c_4, c_8$ $c_9$	$(u^2 + 2)^2$
$c_7$	$(u + 1)^4$
$c_{10}, c_{11}$	$(u - 1)^4$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1, c_2, c_5$ $c_6$	$(y^2 + y + 1)^2$
$c_3, c_4, c_8$ $c_9$	$(y + 2)^4$
$c_7, c_{10}, c_{11}$	$(y - 1)^4$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_2^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.500000 + 0.866025I$ $a = 0.724745 + 0.158919I$ $b = 1.00000$	$3.28987 + 2.02988I$	$-6.00000 - 3.46410I$
$u = 0.500000 + 0.866025I$ $a = -1.72474 + 1.57313I$ $b = 1.00000$	$3.28987 + 2.02988I$	$-6.00000 - 3.46410I$
$u = 0.500000 - 0.866025I$ $a = 0.724745 - 0.158919I$ $b = 1.00000$	$3.28987 - 2.02988I$	$-6.00000 + 3.46410I$
$u = 0.500000 - 0.866025I$ $a = -1.72474 - 1.57313I$ $b = 1.00000$	$3.28987 - 2.02988I$	$-6.00000 + 3.46410I$

$$\text{III. } I_3^u = \langle b + 1, a - u - 1, u^2 + u + 1 \rangle$$

(i) Arc colorings

$$a_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_2 = \begin{pmatrix} 1 \\ u + 1 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} -u \\ u + 1 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -u \\ u + 1 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} u + 1 \\ -1 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} -u \\ u + 1 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} u + 2 \\ -1 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} u + 1 \\ -1 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} u + 1 \\ -1 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} u + 1 \\ -1 \end{pmatrix}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes =  $4u - 10$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1, c_2, c_6$	$u^2 + u + 1$
$c_3, c_4, c_8$ $c_9$	$u^2$
$c_5$	$u^2 - u + 1$
$c_7, c_{10}$	$(u - 1)^2$
$c_{11}$	$(u + 1)^2$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1, c_2, c_5$ $c_6$	$y^2 + y + 1$
$c_3, c_4, c_8$ $c_9$	$y^2$
$c_7, c_{10}, c_{11}$	$(y - 1)^2$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_3^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.500000 + 0.866025I$ $a = 0.500000 + 0.866025I$ $b = -1.00000$	$-1.64493 - 2.02988I$	$-12.00000 + 3.46410I$
$u = -0.500000 - 0.866025I$ $a = 0.500000 - 0.866025I$ $b = -1.00000$	$-1.64493 + 2.02988I$	$-12.00000 - 3.46410I$

#### IV. u-Polynomials

Crossings	u-Polynomials at each crossing
$c_1$	$((u^2 - u + 1)^2)(u^2 + u + 1)(u^{26} - 2u^{25} + \dots - u + 3)$
$c_2$	$((u^2 + u + 1)^3)(u^{26} + 16u^{25} + \dots - 43u + 9)$
$c_3, c_4, c_8$	$u^2(u^2 + 2)^2(u^{26} + u^{25} + \dots - 8u - 4)$
$c_5$	$(u^2 - u + 1)(u^2 + u + 1)^2(u^{26} - 2u^{25} + \dots - u + 3)$
$c_6$	$((u^2 - u + 1)^2)(u^2 + u + 1)(u^{26} + 2u^{25} + \dots - 13u + 3)$
$c_7$	$((u - 1)^2)(u + 1)^4(u^{26} + 3u^{25} + \dots + 22u - 3)$
$c_9$	$u^2(u^2 + 2)^2(u^{26} - u^{25} + \dots - 32u - 4)$
$c_{10}$	$((u - 1)^6)(u^{26} + 33u^{25} + \dots + 64u + 9)$
$c_{11}$	$((u - 1)^4)(u + 1)^2(u^{26} + 3u^{25} + \dots + 22u - 3)$



### V. Riley Polynomials

Crossings	Riley Polynomials at each crossing
$c_1, c_5$	$((y^2 + y + 1)^3)(y^{26} + 16y^{25} + \dots - 43y + 9)$
$c_2$	$((y^2 + y + 1)^3)(y^{26} - 8y^{25} + \dots - 7123y + 81)$
$c_3, c_4, c_8$	$y^2(y + 2)^4(y^{26} + 21y^{25} + \dots + 64y + 16)$
$c_6$	$((y^2 + y + 1)^3)(y^{26} - 32y^{25} + \dots - 187y + 9)$
$c_7, c_{11}$	$((y - 1)^6)(y^{26} - 33y^{25} + \dots - 64y + 9)$
$c_9$	$y^2(y + 2)^4(y^{26} - 39y^{25} + \dots - 128y + 16)$
$c_{10}$	$((y - 1)^6)(y^{26} - 73y^{25} + \dots + 35108y + 81)$