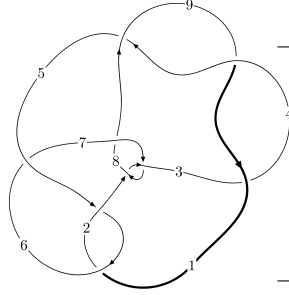
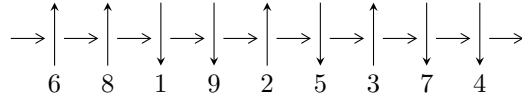


9<sub>37</sub> (K9a<sub>18</sub>)

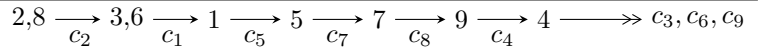


A knot diagram<sup>1</sup>

**Linearized knot diagram**



**Solving Sequence**



**Ideals for irreducible components<sup>2</sup> of  $X_{\text{par}}$**

$$I_1^u = \langle b - u, -u^7 - 2u^3 - u^2 + 2a + u - 1, u^8 - u^7 + 2u^6 - 2u^5 + 4u^4 - 3u^3 + 2u^2 + 1 \rangle$$

$$I_2^u = \langle -u^3 + b - 1, u^5 - u^3 + 2u^2 + 2a + u - 1, u^6 + u^4 + 2u^3 + u^2 + u + 2 \rangle$$

$$I_3^u = \langle -u^3 + b - u + 1, -u^2 + a + u - 1, u^4 - u^3 + 2u^2 - 2u + 1 \rangle$$

$$I_4^u = \langle b - a - u - 1, a^2 + 3au + 2a - 1, u^2 + u + 1 \rangle$$

$$I_5^u = \langle b - u, a - u - 2, u^2 + u + 1 \rangle$$

$$I_6^u = \langle b + u, a + 2u - 1, u^2 + 1 \rangle$$

\* 6 irreducible components of  $\dim_{\mathbb{C}} = 0$ , with total 26 representations.

<sup>1</sup>The image of knot diagram is generated by the software “**Draw programme**” developed by Andrew Bartholomew(<http://www.layer8.co.uk/maths/draw/index.htm#Running-draw>), where we modified some parts for our purpose(<https://github.com/CATsTAILs/LinksPainter>).

<sup>2</sup>All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

I.

$$I_1^u = \langle b-u, -u^7-2u^3-u^2+2a+u-1, u^8-u^7+2u^6-2u^5+4u^4-3u^3+2u^2+1 \rangle$$

(i) Arc colorings

$$a_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_3 = \begin{pmatrix} 1 \\ -u^2 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} \frac{1}{2}u^7 + u^3 + \frac{1}{2}u^2 - \frac{1}{2}u + \frac{1}{2} \\ u \end{pmatrix}$$

$$a_1 = \begin{pmatrix} \frac{1}{2}u^7 - u^6 + \dots + \frac{1}{2}u + \frac{1}{2} \\ u^2 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} \frac{1}{2}u^7 + u^3 + \frac{1}{2}u^2 - \frac{3}{2}u + \frac{1}{2} \\ u \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -u \\ u^3 + u \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -u^3 \\ u^5 + u^3 + u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} u^2 - u + 1 \\ \frac{1}{2}u^7 + u^5 + \dots + \frac{1}{2}u - \frac{1}{2} \end{pmatrix}$$

$$a_4 = \begin{pmatrix} u^2 - u + 1 \\ \frac{1}{2}u^7 + u^5 + \dots + \frac{1}{2}u - \frac{1}{2} \end{pmatrix}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes =  $-4u^6 + 2u^5 - 4u^4 + 6u^3 - 12u^2 + 6u$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1, c_2, c_5$ $c_7$	$u^8 + u^7 + 2u^6 + 2u^5 + 4u^4 + 3u^3 + 2u^2 + 1$
$c_3, c_4, c_9$	$u^8 - 2u^7 + 6u^6 - 8u^5 + 10u^4 - 9u^3 + 5u^2 - 3u + 2$
$c_6, c_8$	$u^8 + 3u^7 + 8u^6 + 10u^5 + 14u^4 + 11u^3 + 12u^2 + 4u + 1$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1, c_2, c_5$ $c_7$	$y^8 + 3y^7 + 8y^6 + 10y^5 + 14y^4 + 11y^3 + 12y^2 + 4y + 1$
$c_3, c_4, c_9$	$y^8 + 8y^7 + 24y^6 + 30y^5 + 8y^4 - 5y^3 + 11y^2 + 11y + 4$
$c_6, c_8$	$y^8 + 7y^7 + 32y^6 + 82y^5 + 146y^4 + 151y^3 + 84y^2 + 8y + 1$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.862697 + 0.615401I$ $a = -0.361509 + 0.665983I$ $b = 0.862697 + 0.615401I$	$7.44069 + 0.66722I$	$4.81639 - 2.10627I$
$u = 0.862697 - 0.615401I$ $a = -0.361509 - 0.665983I$ $b = 0.862697 - 0.615401I$	$7.44069 - 0.66722I$	$4.81639 + 2.10627I$
$u = 0.578102 + 1.055330I$ $a = -1.26281 + 1.67027I$ $b = 0.578102 + 1.055330I$	$-1.73404 + 6.79402I$	$-3.11839 - 7.09473I$
$u = 0.578102 - 1.055330I$ $a = -1.26281 - 1.67027I$ $b = 0.578102 - 1.055330I$	$-1.73404 - 6.79402I$	$-3.11839 + 7.09473I$
$u = -0.666851 + 1.155530I$ $a = 0.88635 + 1.91065I$ $b = -0.666851 + 1.155530I$	$3.94193 - 10.98940I$	$0.47099 + 7.14773I$
$u = -0.666851 - 1.155530I$ $a = 0.88635 - 1.91065I$ $b = -0.666851 - 1.155530I$	$3.94193 + 10.98940I$	$0.47099 - 7.14773I$
$u = -0.273948 + 0.520074I$ $a = 0.737965 - 0.414347I$ $b = -0.273948 + 0.520074I$	$0.221012 - 1.276800I$	$1.83102 + 5.88514I$
$u = -0.273948 - 0.520074I$ $a = 0.737965 + 0.414347I$ $b = -0.273948 - 0.520074I$	$0.221012 + 1.276800I$	$1.83102 - 5.88514I$

$$\text{II. } I_2^u = \langle -u^3 + b - 1, u^5 - u^3 + 2u^2 + 2a + u - 1, u^6 + u^4 + 2u^3 + u^2 + u + 2 \rangle$$

(i) Arc colorings

$$a_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_3 = \begin{pmatrix} 1 \\ -u^2 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -\frac{1}{2}u^5 + \frac{1}{2}u^3 - u^2 - \frac{1}{2}u + \frac{1}{2} \\ u^3 + 1 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -\frac{1}{2}u^5 - u^4 + \dots - \frac{3}{2}u - \frac{1}{2} \\ -u^4 - u^2 - u - 1 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -\frac{1}{2}u^5 - \frac{1}{2}u^3 - u^2 - \frac{1}{2}u - \frac{1}{2} \\ u^3 + 1 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -u \\ u^3 + u \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -u^3 \\ u^5 + u^3 + u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} -\frac{1}{2}u^5 + \frac{1}{2}u^3 + \frac{1}{2}u + \frac{3}{2} \\ -u^5 - u^3 - 2u^2 - 1 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} -\frac{1}{2}u^5 + \frac{1}{2}u^3 + \frac{1}{2}u + \frac{3}{2} \\ -u^5 - u^3 - 2u^2 - 1 \end{pmatrix}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes =  $-4u^4 - 4u^3 - 8u - 6$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1, c_2, c_5$ $c_7$	$u^6 + u^4 - 2u^3 + u^2 - u + 2$
$c_3, c_4, c_9$	$(u^3 + 2u - 1)^2$
$c_6, c_8$	$u^6 + 2u^5 + 3u^4 + 2u^3 + u^2 + 3u + 4$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1, c_2, c_5$ $c_7$	$y^6 + 2y^5 + 3y^4 + 2y^3 + y^2 + 3y + 4$
$c_3, c_4, c_9$	$(y^3 + 4y^2 + 4y - 1)^2$
$c_6, c_8$	$y^6 + 2y^5 + 3y^4 - 2y^3 + 13y^2 - y + 16$



(vi) Complex Volumes and Cusp Shapes

Solutions to $I_2^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.931903 + 0.428993I$ $a = -0.180233 + 0.631115I$ $b = 0.705204 + 1.038720I$	$6.15087 + 5.13794I$	$3.31793 - 3.20902I$
$u = -0.931903 - 0.428993I$ $a = -0.180233 - 0.631115I$ $b = 0.705204 - 1.038720I$	$6.15087 - 5.13794I$	$3.31793 + 3.20902I$
$u = 0.226699 + 1.074330I$ $a = 0.41474 - 1.96546I$ $b = 0.226699 - 1.074330I$	$-4.07707$	$-8.63587 + 0.I$
$u = 0.226699 - 1.074330I$ $a = 0.41474 + 1.96546I$ $b = 0.226699 + 1.074330I$	$-4.07707$	$-8.63587 + 0.I$
$u = 0.705204 + 1.038720I$ $a = -0.484509 - 0.229988I$ $b = -0.931903 + 0.428993I$	$6.15087 + 5.13794I$	$3.31793 - 3.20902I$
$u = 0.705204 - 1.038720I$ $a = -0.484509 + 0.229988I$ $b = -0.931903 - 0.428993I$	$6.15087 - 5.13794I$	$3.31793 + 3.20902I$

$$\text{III. } I_3^u = \langle -u^3 + b - u + 1, -u^2 + a + u - 1, u^4 - u^3 + 2u^2 - 2u + 1 \rangle$$

(i) Arc colorings

$$a_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_3 = \begin{pmatrix} 1 \\ -u^2 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} u^2 - u + 1 \\ u^3 + u - 1 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} u \\ -u^3 - u \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -u^3 + u^2 - 2u + 2 \\ u^3 + u - 1 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -u \\ u^3 + u \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -u^3 \\ 2u - 1 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} -u^2 + 1 \\ u^3 - 2u^2 + 2u - 1 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} -u^2 + 1 \\ u^3 - 2u^2 + 2u - 1 \end{pmatrix}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes =  $4u^3 + 4u - 2$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1, c_5$	$(u^2 - u + 1)^2$
$c_2, c_3, c_4$ $c_7, c_9$	$u^4 + u^3 + 2u^2 + 2u + 1$
$c_6$	$(u^2 + u + 1)^2$
$c_8$	$u^4 + 3u^3 + 2u^2 + 1$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1, c_5, c_6$	$(y^2 + y + 1)^2$
$c_2, c_3, c_4$ $c_7, c_9$	$y^4 + 3y^3 + 2y^2 + 1$
$c_8$	$y^4 - 5y^3 + 6y^2 + 4y + 1$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_3^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.621744 + 0.440597I$	$- 2.02988I$	$0. + 3.46410I$
$a = 0.570696 + 0.107280I$		
$b = -0.500000 + 0.866025I$		
$u = 0.621744 - 0.440597I$	$2.02988I$	$0. - 3.46410I$
$a = 0.570696 - 0.107280I$		
$b = -0.500000 - 0.866025I$		
$u = -0.121744 + 1.306620I$	$2.02988I$	$0. - 3.46410I$
$a = -0.57070 - 1.62477I$		
$b = -0.500000 - 0.866025I$		
$u = -0.121744 - 1.306620I$	$- 2.02988I$	$0. + 3.46410I$
$a = -0.57070 + 1.62477I$		
$b = -0.500000 + 0.866025I$		

$$\text{IV. } I_4^u = \langle b - a - u - 1, a^2 + 3au + 2a - 1, u^2 + u + 1 \rangle$$

(i) Arc colorings

$$a_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_3 = \begin{pmatrix} 1 \\ u + 1 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} a \\ a + u + 1 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -2au - a + 2 \\ -au + u + 1 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -u - 1 \\ a + u + 1 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -u \\ u + 1 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} a \\ a + u + 1 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} a \\ a + u + 1 \end{pmatrix}$$

(ii) Obstruction class =  $-1$

(iii) Cusp Shapes =  $4u + 2$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1, c_3, c_4$ $c_5, c_9$	$u^4 + u^3 + 2u^2 + 2u + 1$
$c_2, c_7$	$(u^2 - u + 1)^2$
$c_6$	$u^4 + 3u^3 + 2u^2 + 1$
$c_8$	$(u^2 + u + 1)^2$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1, c_3, c_4$ $c_5, c_9$	$y^4 + 3y^3 + 2y^2 + 1$
$c_2, c_7, c_8$	$(y^2 + y + 1)^2$
$c_6$	$y^4 - 5y^3 + 6y^2 + 4y + 1$



(vi) Complex Volumes and Cusp Shapes

Solutions to $I_4^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.500000 + 0.866025I$		
$a = 0.121744 - 0.425428I$	$- 2.02988I$	$0. + 3.46410I$
$b = 0.621744 + 0.440597I$		
$u = -0.500000 + 0.866025I$		
$a = -0.62174 - 2.17265I$	$- 2.02988I$	$0. + 3.46410I$
$b = -0.121744 - 1.306620I$		
$u = -0.500000 - 0.866025I$		
$a = 0.121744 + 0.425428I$	$2.02988I$	$0. - 3.46410I$
$b = 0.621744 - 0.440597I$		
$u = -0.500000 - 0.866025I$		
$a = -0.62174 + 2.17265I$	$2.02988I$	$0. - 3.46410I$
$b = -0.121744 + 1.306620I$		

$$\mathbf{V. } I_5^u = \langle b - u, a - u - 2, u^2 + u + 1 \rangle$$

**(i) Arc colorings**

$$a_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_3 = \begin{pmatrix} 1 \\ u + 1 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} u + 2 \\ u \end{pmatrix}$$

$$a_1 = \begin{pmatrix} u \\ -u - 1 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} 2 \\ u \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -u \\ u + 1 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} u + 2 \\ u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} u + 2 \\ u \end{pmatrix}$$

**(ii) Obstruction class = -1**

**(iii) Cusp Shapes =  $4u + 2$**

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1, c_2, c_3$ $c_4, c_5, c_7$ $c_9$	$u^2 - u + 1$
$c_6, c_8$	$u^2 + u + 1$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1, c_2, c_3$	$y^2 + y + 1$
$c_4, c_5, c_6$	
$c_7, c_8, c_9$	

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_5^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.500000 + 0.866025I$	$-2.02988I$	$0. + 3.46410I$
$a = 1.50000 + 0.86603I$		
$b = -0.500000 + 0.866025I$		
$u = -0.500000 - 0.866025I$	$2.02988I$	$0. - 3.46410I$
$a = 1.50000 - 0.86603I$		
$b = -0.500000 - 0.866025I$		

$$\text{VI. } I_6^u = \langle b + u, a + 2u - 1, u^2 + 1 \rangle$$

(i) Arc colorings

$$a_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_3 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -2u + 1 \\ -u \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -u - 1 \\ -1 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -u + 1 \\ -u \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -u \\ 0 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} u \\ u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} -u + 2 \\ -u + 1 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} -u + 2 \\ -u + 1 \end{pmatrix}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes = -4

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1, c_2, c_3$ $c_4, c_5, c_7$ $c_9$	$u^2 + 1$
$c_6, c_8$	$(u + 1)^2$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1, c_2, c_3$ $c_4, c_5, c_7$ $c_9$	$(y + 1)^2$
$c_6, c_8$	$(y - 1)^2$



(vi) Complex Volumes and Cusp Shapes

Solutions to $I_6^u$		$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u =$	$1.000000I$		
$a =$	$1.00000 - 2.00000I$	$-1.64493$	$-4.00000$
$b =$	$-1.000000I$		
$u =$	$-1.000000I$		
$a =$	$1.00000 + 2.00000I$	$-1.64493$	$-4.00000$
$b =$	$1.000000I$		

## VII. u-Polynomials

Crossings	u-Polynomials at each crossing
$c_1, c_2, c_5$ $c_7$	$(u^2 + 1)(u^2 - u + 1)^3(u^4 + u^3 + \dots + 2u + 1)(u^6 + u^4 + \dots - u + 2)$ $\cdot (u^8 + u^7 + 2u^6 + 2u^5 + 4u^4 + 3u^3 + 2u^2 + 1)$
$c_3, c_4, c_9$	$(u^2 + 1)(u^2 - u + 1)(u^3 + 2u - 1)^2(u^4 + u^3 + 2u^2 + 2u + 1)^2$ $\cdot (u^8 - 2u^7 + 6u^6 - 8u^5 + 10u^4 - 9u^3 + 5u^2 - 3u + 2)$
$c_6, c_8$	$(u + 1)^2(u^2 + u + 1)^3(u^4 + 3u^3 + 2u^2 + 1)$ $\cdot (u^6 + 2u^5 + 3u^4 + 2u^3 + u^2 + 3u + 4)$ $\cdot (u^8 + 3u^7 + 8u^6 + 10u^5 + 14u^4 + 11u^3 + 12u^2 + 4u + 1)$

### VIII. Riley Polynomials

Crossings	Riley Polynomials at each crossing
$c_1, c_2, c_5$ $c_7$	$(y + 1)^2(y^2 + y + 1)^3(y^4 + 3y^3 + 2y^2 + 1)$ $\cdot (y^6 + 2y^5 + 3y^4 + 2y^3 + y^2 + 3y + 4)$ $\cdot (y^8 + 3y^7 + 8y^6 + 10y^5 + 14y^4 + 11y^3 + 12y^2 + 4y + 1)$
$c_3, c_4, c_9$	$(y + 1)^2(y^2 + y + 1)(y^3 + 4y^2 + 4y - 1)^2(y^4 + 3y^3 + 2y^2 + 1)^2$ $\cdot (y^8 + 8y^7 + 24y^6 + 30y^5 + 8y^4 - 5y^3 + 11y^2 + 11y + 4)$
$c_6, c_8$	$(y - 1)^2(y^2 + y + 1)^3(y^4 - 5y^3 + 6y^2 + 4y + 1)$ $\cdot (y^6 + 2y^5 + 3y^4 - 2y^3 + 13y^2 - y + 16)$ $\cdot (y^8 + 7y^7 + 32y^6 + 82y^5 + 146y^4 + 151y^3 + 84y^2 + 8y + 1)$