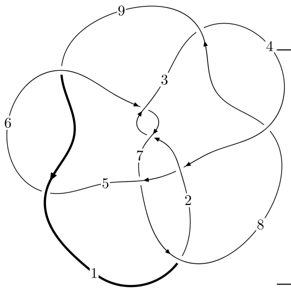
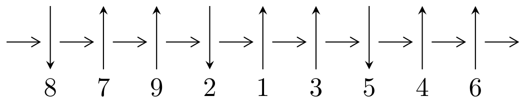
9_{41} $(K9a_{29})$



A knot diagram[1]

Linearized knot diagam



Solving Sequence

$$3,9 \xrightarrow[c_3]{} 4,6 \xrightarrow[c_6]{} 7 \xrightarrow[c_9]{} 1 \xrightarrow[c_2]{} 2 \xrightarrow[c_5]{} 5 \xrightarrow[c_8]{} 8 \twoheadrightarrow c_1, c_4, c_7$$

Ideals for irreducible components[2]of $X_{\mathbf{par}}$

$$I_1^u = \langle b+u,\ a-1,\ u^4-2u^3+4u^2-3u+1\rangle$$
$$I_2^u = \langle b+u,\ a^2+au+2u^2+2a+2u+4,\ u^3+u^2+2u+1\rangle$$
$$I_3^u = \langle -3u^5+11u^4-26u^3+35u^2+4b-28u+12,\ 3u^5-9u^4+20u^3-23u^2+8a+14u-4,$$
$$u^6-5u^5+14u^4-25u^3+28u^2-20u+8\rangle$$
$$I_4^u = \langle -u^2b+b^2-2bu-2u,\ a-1,\ u^3+u^2+2u+1\rangle$$
$$I_5^u = \langle b+u,\ a+1,\ u^4+2u^2-u+1\rangle$$
$$I_6^u = \langle -au+b-u-1,\ -u^2a+a^2-au+3u^2-a+u+5,\ u^3+u^2+2u+1\rangle$$

* 6 irreducible components of $\dim_{\mathbb{C}} = 0$, with total 32 representations.

[1]The image of knot diagram is generated by the software "**Draw programme**" developed by Andrew Bartholomew(`http://www.layer8.co.uk/maths/draw/index.htm#Running-draw`), where we modified some parts for our purpose(`https://github.com/CATsTAILs/LinksPainter`).

[2]All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

I. $\boldsymbol{I_1^u = \langle b+u,\ a-1,\ u^4-2u^3+4u^2-3u+1\rangle}$

(i) Arc colorings

$$a_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 1 \\ -u^2 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} 1 \\ -u \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -u+1 \\ -u \end{pmatrix}$$

$$a_1 = \begin{pmatrix} u \\ -u^2+u \end{pmatrix}$$

$$a_2 = \begin{pmatrix} u^2-u+1 \\ u^2 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} u^2+1 \\ -u^3+u^2-u \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -u \\ u^3+u \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -u \\ u^3+u \end{pmatrix}$$

(ii) Obstruction class $= -1$

(iii) Cusp Shapes $= -3u^3+9u^2-18u+15$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
c_1, c_4, c_7	$u^4 - 3u^3 + 4u^2 - 2u + 1$
c_2, c_3, c_5 c_6, c_8, c_9	$u^4 - 2u^3 + 4u^2 - 3u + 1$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
c_1, c_4, c_7	$y^4 - y^3 + 6y^2 + 4y + 1$
c_2, c_3, c_5 c_6, c_8, c_9	$y^4 + 4y^3 + 6y^2 - y + 1$

(vi) Complex Volumes and Cusp Shapes

Solutions to I_1^u	$\sqrt{-1}(\mathrm{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.500000 + 0.363271I$ $a = 1.00000$ $b = -0.500000 - 0.363271I$	$0.986960 + 0.735995I$	$7.28115 - 3.94298I$
$u = 0.500000 - 0.363271I$ $a = 1.00000$ $b = -0.500000 + 0.363271I$	$0.986960 - 0.735995I$	$7.28115 + 3.94298I$
$u = 0.50000 + 1.53884I$ $a = 1.00000$ $b = -0.50000 - 1.53884I$	$-10.8566 + 12.0989I$	$-2.78115 - 6.37988I$
$u = 0.50000 - 1.53884I$ $a = 1.00000$ $b = -0.50000 + 1.53884I$	$-10.8566 - 12.0989I$	$-2.78115 + 6.37988I$

II. $I_2^u = \langle b+u,\ a^2+au+2u^2+2a+2u+4,\ u^3+u^2+2u+1\rangle$

(i) Arc colorings

$$a_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$
$$a_9 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$
$$a_4 = \begin{pmatrix} 1 \\ -u^2 \end{pmatrix}$$
$$a_6 = \begin{pmatrix} a \\ -u \end{pmatrix}$$
$$a_7 = \begin{pmatrix} a-u \\ -u \end{pmatrix}$$
$$a_1 = \begin{pmatrix} -u^2a-2au+2 \\ -u^2a+u \end{pmatrix}$$
$$a_2 = \begin{pmatrix} -au+u^2+1 \\ u^2 \end{pmatrix}$$
$$a_5 = \begin{pmatrix} -u^2a-5au-2u^2-2a-4u \\ -2u^2a-3au-2u^2-a-u \end{pmatrix}$$
$$a_8 = \begin{pmatrix} -u \\ -u^2-u-1 \end{pmatrix}$$
$$a_8 = \begin{pmatrix} -u \\ -u^2-u-1 \end{pmatrix}$$

(ii) Obstruction class $= -1$

(iii) Cusp Shapes $= 4au+8u^2+4a+12u+10$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
c_1, c_7	$(u^3+u^2-1)^2$
c_2, c_3, c_6 c_8	$(u^3+u^2+2u+1)^2$
c_4	$u^6-7u^5+24u^4-47u^3+54u^2-32u+8$
c_5, c_9	$u^6-5u^5+14u^4-25u^3+28u^2-20u+8$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
c_1, c_7	$(y^3 - y^2 + 2y - 1)^2$
c_2, c_3, c_6 c_8	$(y^3 + 3y^2 + 2y - 1)^2$
c_4	$y^6 - y^5 + 26y^4 - 49y^3 + 292y^2 - 160y + 64$
c_5, c_9	$y^6 + 3y^5 + 2y^4 - 25y^3 + 8y^2 + 48y + 64$

(vi) Complex Volumes and Cusp Shapes

Solutions to I_2^u	$\sqrt{-1}(\mathrm{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.215080 + 1.307140I$ $a = -0.407481 - 0.986732I$ $b = \ \ 0.215080 - 1.307140I$	$-4.93480 - 5.65624I$	$-2.00000 + 5.95889I$
$u = -0.215080 + 1.307140I$ $a = -1.37744 - 0.32041I$ $b = \ \ 0.215080 - 1.307140I$	$-9.07239 - 2.82812I$	$-8.52927 + 2.97945I$
$u = -0.215080 - 1.307140I$ $a = -0.407481 + 0.986732I$ $b = \ \ 0.215080 + 1.307140I$	$-4.93480 + 5.65624I$	$-2.00000 - 5.95889I$
$u = -0.215080 - 1.307140I$ $a = -1.37744 + 0.32041I$ $b = \ \ 0.215080 + 1.307140I$	$-9.07239 + 2.82812I$	$-8.52927 - 2.97945I$
$u = -0.569840$ $a = -0.71508 + 1.73159I$ $b = \ \ 0.569840$	$-0.79722 - 2.82812I$	$4.52927 + 2.97945I$
$u = -0.569840$ $a = -0.71508 - 1.73159I$ $b = \ \ 0.569840$	$-0.79722 + 2.82812I$	$4.52927 - 2.97945I$

III. $\boldsymbol{I_3^u = \langle -3u^5 + 11u^4 + \cdots + 4b + 12,\ 3u^5 - 9u^4 + 20u^3 - 23u^2 + 8a + 14u - 4,\ u^6 - 5u^5 + 14u^4 - 25u^3 + 28u^2 - 20u + 8\rangle}$

(i) Arc colorings

$$a_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 1 \\ -u^2 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -\frac{3}{8}u^5 + \frac{9}{8}u^4 + \cdots - \frac{7}{4}u + \frac{1}{2} \\ \frac{3}{4}u^5 - \frac{11}{4}u^4 + \cdots + 7u - 3 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} \frac{3}{8}u^5 - \frac{13}{8}u^4 + \cdots + \frac{21}{4}u - \frac{5}{2} \\ \frac{3}{4}u^5 - \frac{11}{4}u^4 + \cdots + 7u - 3 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} \frac{5}{8}u^5 - \frac{19}{8}u^4 + \cdots + \frac{23}{4}u - 3 \\ -\frac{3}{4}u^5 + \frac{13}{4}u^4 + \cdots - \frac{17}{2}u + 5 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} \frac{1}{8}u^5 - \frac{7}{8}u^4 + \cdots + \frac{15}{4}u - 1 \\ \frac{3}{4}u^5 - \frac{13}{4}u^4 + \cdots + \frac{19}{2}u - 5 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -\frac{1}{8}u^5 + \frac{5}{8}u^4 + \cdots + \frac{13}{8}u^2 - \frac{1}{2}u \\ -u^5 + \frac{7}{2}u^4 - \frac{17}{2}u^3 + 11u^2 - \frac{19}{2}u + 5 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -u \\ u^3 + u \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -u \\ u^3 + u \end{pmatrix}$$

(ii) Obstruction class $= -1$

(iii) Cusp Shapes $= -u^5 + u^4 - 2u^3 + u^2 - 4u + 6$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
c_1, c_4	$(u^3+u^2-1)^2$
c_2, c_5, c_6 c_9	$(u^3+u^2+2u+1)^2$
c_3, c_8	$u^6-5u^5+14u^4-25u^3+28u^2-20u+8$
c_7	$u^6-7u^5+24u^4-47u^3+54u^2-32u+8$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
c_1, c_4	$(y^3 - y^2 + 2y - 1)^2$
c_2, c_5, c_6 c_9	$(y^3 + 3y^2 + 2y - 1)^2$
c_3, c_8	$y^6 + 3y^5 + 2y^4 - 25y^3 + 8y^2 + 48y + 64$
c_7	$y^6 - y^5 + 26y^4 - 49y^3 + 292y^2 - 160y + 64$

(vi) Complex Volumes and Cusp Shapes

Solutions to I_3^u	$\sqrt{-1}(\mathrm{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.407481 + 0.986732I$ $a = -0.203741 + 0.493366I$ $b = 0.569840$	$-0.79722 + 2.82812I$	$4.52927 - 2.97945I$
$u = 0.407481 - 0.986732I$ $a = -0.203741 - 0.493366I$ $b = 0.569840$	$-0.79722 - 2.82812I$	$4.52927 + 2.97945I$
$u = 1.37744 + 0.32041I$ $a = -0.357540 - 0.865797I$ $b = 0.215080 + 1.307140I$	$-4.93480 + 5.65624I$	$-2.00000 - 5.95889I$
$u = 1.37744 - 0.32041I$ $a = -0.357540 + 0.865797I$ $b = 0.215080 - 1.307140I$	$-4.93480 - 5.65624I$	$-2.00000 + 5.95889I$
$u = 0.71508 + 1.73159I$ $a = -0.688719 - 0.160205I$ $b = 0.215080 + 1.307140I$	$-9.07239 + 2.82812I$	$-8.52927 - 2.97945I$
$u = 0.71508 - 1.73159I$ $a = -0.688719 + 0.160205I$ $b = 0.215080 - 1.307140I$	$-9.07239 - 2.82812I$	$-8.52927 + 2.97945I$

IV. $\boldsymbol{I_4^u = \langle -u^2b + b^2 - 2bu - 2u,\ a - 1,\ u^3 + u^2 + 2u + 1\rangle}$

(i) Arc colorings

$$a_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$
$$a_9 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$
$$a_4 = \begin{pmatrix} 1 \\ -u^2 \end{pmatrix}$$
$$a_6 = \begin{pmatrix} 1 \\ b \end{pmatrix}$$
$$a_7 = \begin{pmatrix} b+1 \\ b \end{pmatrix}$$
$$a_1 = \begin{pmatrix} u \\ bu+u \end{pmatrix}$$
$$a_2 = \begin{pmatrix} u^2b + 2bu + b + 2u + 1 \\ u^2b + 2bu + 2u \end{pmatrix}$$
$$a_5 = \begin{pmatrix} u^2 + 1 \\ u^2b + u^2 + b \end{pmatrix}$$
$$a_8 = \begin{pmatrix} -u \\ -u^2 - u - 1 \end{pmatrix}$$
$$a_8 = \begin{pmatrix} -u \\ -u^2 - u - 1 \end{pmatrix}$$

(ii) Obstruction class $= -1$

(iii) Cusp Shapes $= 4u^2b + 4bu + 8u^2 + 4b + 12u + 10$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
c_1	$u^6-7u^5+24u^4-47u^3+54u^2-32u+8$
c_2, c_6	$u^6-5u^5+14u^4-25u^3+28u^2-20u+8$
c_3, c_5, c_8 c_9	$(u^3+u^2+2u+1)^2$
c_4, c_7	$(u^3+u^2-1)^2$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
c_1	$y^6 - y^5 + 26y^4 - 49y^3 + 292y^2 - 160y + 64$
c_2, c_6	$y^6 + 3y^5 + 2y^4 - 25y^3 + 8y^2 + 48y + 64$
c_3, c_5, c_8 c_9	$(y^3 + 3y^2 + 2y - 1)^2$
c_4, c_7	$(y^3 - y^2 + 2y - 1)^2$

(vi) Complex Volumes and Cusp Shapes

Solutions to I_4^u	$\sqrt{-1}(\mathrm{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.215080 + 1.307140I$ $a = \quad 1.00000$ $b = -1.37744 + 0.32041I$	$-4.93480 - 5.65624I$	$-2.00000 + 5.95889I$
$u = -0.215080 + 1.307140I$ $a = \quad 1.00000$ $b = -0.71508 + 1.73159I$	$-9.07239 - 2.82812I$	$-8.52927 + 2.97945I$
$u = -0.215080 - 1.307140I$ $a = \quad 1.00000$ $b = -1.37744 - 0.32041I$	$-4.93480 + 5.65624I$	$-2.00000 - 5.95889I$
$u = -0.215080 - 1.307140I$ $a = \quad 1.00000$ $b = -0.71508 - 1.73159I$	$-9.07239 + 2.82812I$	$-8.52927 - 2.97945I$
$u = -0.569840$ $a = \quad 1.00000$ $b = -0.407481 + 0.986732I$	$-0.79722 - 2.82812I$	$4.52927 + 2.97945I$
$u = -0.569840$ $a = \quad 1.00000$ $b = -0.407481 - 0.986732I$	$-0.79722 + 2.82812I$	$4.52927 - 2.97945I$

V. $\boldsymbol{I^u_5 = \langle b + u,\ a + 1,\ u^4 + 2u^2 - u + 1\rangle}$

(i) Arc colorings

$$a_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 1 \\ -u^2 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -1 \\ -u \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -u - 1 \\ -u \end{pmatrix}$$

$$a_1 = \begin{pmatrix} u \\ u^2 + u \end{pmatrix}$$

$$a_2 = \begin{pmatrix} u^2 + u + 1 \\ u^2 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -u^2 - 1 \\ -u^3 - u^2 - u \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -u \\ u^3 + u \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -u \\ u^3 + u \end{pmatrix}$$

(ii) Obstruction class $= 1$

(iii) Cusp Shapes $= -3u^3 - 3u^2 - 6u - 3$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
c_1, c_4, c_7	$u^4 + u^3 + 1$
c_2, c_5, c_8	$u^4 + 2u^2 + u + 1$
c_3, c_6, c_9	$u^4 + 2u^2 - u + 1$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
c_1, c_4, c_7	$y^4 - y^3 + 2y^2 + 1$
c_2, c_3, c_5 c_6, c_8, c_9	$y^4 + 4y^3 + 6y^2 + 3y + 1$

(vi) Complex Volumes and Cusp Shapes

Solutions to I_5^u	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.343815 + 0.625358I$ $a = -1.00000$ $b = -0.343815 - 0.625358I$	$-2.15173 + 3.38562I$	$-3.15611 - 4.97381I$
$u = 0.343815 - 0.625358I$ $a = -1.00000$ $b = -0.343815 + 0.625358I$	$-2.15173 - 3.38562I$	$-3.15611 + 4.97381I$
$u = -0.343815 + 1.358440I$ $a = -1.00000$ $b = 0.343815 - 1.358440I$	$-7.71788 - 2.37936I$	$-1.34389 + 0.72682I$
$u = -0.343815 - 1.358440I$ $a = -1.00000$ $b = 0.343815 + 1.358440I$	$-7.71788 + 2.37936I$	$-1.34389 - 0.72682I$

VI.

$$\boldsymbol{I_6^u = \langle -au + b - u - 1,\ -u^2a + a^2 - au + 3u^2 - a + u + 5,\ u^3 + u^2 + 2u + 1\rangle}$$

(i) Arc colorings

$$a_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 1 \\ -u^2 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} a \\ au + u + 1 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} au + a + u + 1 \\ au + u + 1 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -au + 2u^2 - a + u + 3 \\ -u^2 - 2 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} u^2a + au + 2u^2 + 2u + 3 \\ u^2a + au + u - 1 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} au - u^2 - 1 \\ u^2a + au + a + u + 1 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -u \\ -u^2 - u - 1 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -u \\ -u^2 - u - 1 \end{pmatrix}$$

(ii) Obstruction class $= -1$

(iii) Cusp Shapes $= -2$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
c_1, c_4, c_7	$(u^3 + u^2 - 1)^2$
c_2, c_3, c_5 c_6, c_8, c_9	$(u^3 + u^2 + 2u + 1)^2$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
c_1, c_4, c_7	$(y^3 - y^2 + 2y - 1)^2$
c_2, c_3, c_5 c_6, c_8, c_9	$(y^3 + 3y^2 + 2y - 1)^2$

(vi) Complex Volumes and Cusp Shapes

Solutions to I^u_6	$\sqrt{-1}(\mathrm{vol}+\sqrt{-1}CS)$	Cusp shape
$u = -0.215080 + 1.307140I$ $a = -0.947279 + 0.320410I$ $b = 0.569840$	-4.93480	-2.00000
$u = -0.215080 + 1.307140I$ $a = 0.069840 + 0.424452I$ $b = 0.215080 + 1.307140I$	-4.93480	-2.00000
$u = -0.215080 - 1.307140I$ $a = -0.947279 - 0.320410I$ $b = 0.569840$	-4.93480	-2.00000
$u = -0.215080 - 1.307140I$ $a = 0.069840 - 0.424452I$ $b = 0.215080 - 1.307140I$	-4.93480	-2.00000
$u = -0.569840$ $a = 0.37744 + 2.29387I$ $b = 0.215080 - 1.307140I$	-4.93480	-2.00000
$u = -0.569840$ $a = 0.37744 - 2.29387I$ $b = 0.215080 + 1.307140I$	-4.93480	-2.00000

VII. u-Polynomials

Crossings	u-Polynomials at each crossing
c_1, c_4, c_7	$(u^3+u^2-1)^6(u^4-3u^3+4u^2-2u+1)(u^4+u^3+1)$ $\cdot(u^6-7u^5+24u^4-47u^3+54u^2-32u+8)$
c_2, c_5, c_8	$(u^3+u^2+2u+1)^6(u^4+2u^2+u+1)(u^4-2u^3+4u^2-3u+1)$ $\cdot(u^6-5u^5+14u^4-25u^3+28u^2-20u+8)$
c_3, c_6, c_9	$(u^3+u^2+2u+1)^6(u^4+2u^2-u+1)(u^4-2u^3+4u^2-3u+1)$ $\cdot(u^6-5u^5+14u^4-25u^3+28u^2-20u+8)$

VIII. Riley Polynomials

Crossings	Riley Polynomials at each crossing
c_1, c_4, c_7	$(y^3 - y^2 + 2y - 1)^6(y^4 - y^3 + 2y^2 + 1)(y^4 - y^3 + 6y^2 + 4y + 1)$ $\cdot (y^6 - y^5 + 26y^4 - 49y^3 + 292y^2 - 160y + 64)$
c_2, c_3, c_5 c_6, c_8, c_9	$((y^3 + 3y^2 + 2y - 1)^6)(y^4 + 4y^3 + 6y^2 - y + 1)(y^4 + 4y^3 + \cdots + 3y + 1)$ $\cdot (y^6 + 3y^5 + 2y^4 - 25y^3 + 8y^2 + 48y + 64)$