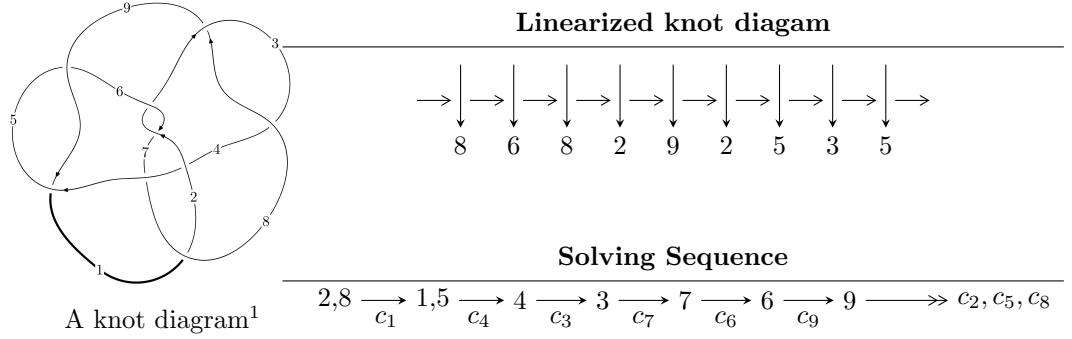


9<sub>49</sub> (K9n<sub>8</sub>)



Ideals for irreducible components<sup>2</sup> of  $X_{\text{par}}$

$$\begin{aligned} I_1^u &= \langle b + u, a - 1, u^3 - 3u^2 + 2u + 1 \rangle \\ I_2^u &= \langle b + u, a^2 - au + 2u + 4, u^2 + u - 1 \rangle \\ I_3^u &= \langle u^3 - 3u^2 + 2b + 3u - 4, -u^3 + 2u^2 + 2a - 2u + 3, u^4 - 3u^3 + 5u^2 - 6u + 4 \rangle \\ I_4^u &= \langle b^2 - bu + b + 2, a - 1, u^2 + u - 1 \rangle \\ I_5^u &= \langle b + u, a + 1, u^3 + u^2 + 1 \rangle \end{aligned}$$

\* 5 irreducible components of  $\dim_{\mathbb{C}} = 0$ , with total 18 representations.

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<sup>1</sup>The image of knot diagram is generated by the software “**Draw programme**” developed by Andrew Bartholomew(<http://www.layer8.co.uk/math/draw/index.htm#Running-draw>), where we modified some parts for our purpose(<https://github.com/CATsTAILs/LinksPainter>).

<sup>2</sup>All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

$$\text{I. } I_1^u = \langle b + u, a - 1, u^3 - 3u^2 + 2u + 1 \rangle$$

(i) Arc colorings

$$\begin{aligned} a_2 &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ a_8 &= \begin{pmatrix} 0 \\ u \end{pmatrix} \\ a_1 &= \begin{pmatrix} 1 \\ -u^2 \end{pmatrix} \\ a_5 &= \begin{pmatrix} 1 \\ -u \end{pmatrix} \\ a_4 &= \begin{pmatrix} -u + 1 \\ -u \end{pmatrix} \\ a_3 &= \begin{pmatrix} -u + 1 \\ 2u^2 - 3u - 1 \end{pmatrix} \\ a_7 &= \begin{pmatrix} u \\ -u^2 + u \end{pmatrix} \\ a_6 &= \begin{pmatrix} -u^2 + 2u \\ -u^2 + u \end{pmatrix} \\ a_9 &= \begin{pmatrix} -u^2 + u + 1 \\ u^2 - 2u - 1 \end{pmatrix} \\ a_9 &= \begin{pmatrix} -u^2 + u + 1 \\ u^2 - 2u - 1 \end{pmatrix} \end{aligned}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes =  $3u^2 - 15$

**(iv) u-Polynomials at the component**

Crossings	u-Polynomials at each crossing
$c_1, c_4, c_7$	$u^3 - 3u^2 + 2u + 1$
$c_2, c_3, c_5$ $c_6, c_8, c_9$	$u^3 + 2u^2 + 3u + 1$

**(v) Riley Polynomials at the component**

Crossings	Riley Polynomials at each crossing
$c_1, c_4, c_7$	$y^3 - 5y^2 + 10y - 1$
$c_2, c_3, c_5$ $c_6, c_8, c_9$	$y^3 + 2y^2 + 5y - 1$

**(vi) Complex Volumes and Cusp Shapes**

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.324718$		
$a = 1.00000$	-0.674976	-14.6840
$b = 0.324718$		
$u = 1.66236 + 0.56228I$		
$a = 1.00000$	$-1.30745 - 9.42707I$	$-7.65816 + 5.60826I$
$b = -1.66236 - 0.56228I$		
$u = 1.66236 - 0.56228I$		
$a = 1.00000$	$-1.30745 + 9.42707I$	$-7.65816 - 5.60826I$
$b = -1.66236 + 0.56228I$		

$$\text{II. } I_2^u = \langle b + u, a^2 - au + 2u + 4, u^2 + u - 1 \rangle$$

(i) Arc colorings

$$a_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_1 = \begin{pmatrix} 1 \\ u-1 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} a \\ -u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} a-u \\ -u \end{pmatrix}$$

$$a_3 = \begin{pmatrix} a-u \\ au-a+u-1 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -au+a-2u-2 \\ au-a+u \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -u-2 \\ au-a+u \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -au+a+3 \\ 2au-a+2u-2 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -au+a+3 \\ 2au-a+2u-2 \end{pmatrix}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes =  $4au - 4a + 4u - 10$

**(iv) u-Polynomials at the component**

Crossings	u-Polynomials at each crossing
$c_1, c_4$	$(u^2 + u - 1)^2$
$c_2, c_5, c_6$ $c_9$	$(u^2 - u + 1)^2$
$c_3, c_8$	$u^4 + 3u^3 + 5u^2 + 6u + 4$
$c_7$	$u^4 - 3u^3 + 5u^2 - 6u + 4$

**(v) Riley Polynomials at the component**

Crossings	Riley Polynomials at each crossing
$c_1, c_4$	$(y^2 - 3y + 1)^2$
$c_2, c_5, c_6$ $c_9$	$(y^2 + y + 1)^2$
$c_3, c_7, c_8$	$y^4 + y^3 - 3y^2 + 4y + 16$

**(vi) Complex Volumes and Cusp Shapes**

Solutions to $I_2^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.618034$		
$a = 0.30902 + 2.26728I$	$3.94784 + 2.02988I$	$-8.00000 - 3.46410I$
$b = -0.618034$		
$u = 0.618034$		
$a = 0.30902 - 2.26728I$	$3.94784 - 2.02988I$	$-8.00000 + 3.46410I$
$b = -0.618034$		
$u = -1.61803$		
$a = -0.809017 + 0.330792I$	$-3.94784 + 2.02988I$	$-8.00000 - 3.46410I$
$b = 1.61803$		
$u = -1.61803$		
$a = -0.809017 - 0.330792I$	$-3.94784 - 2.02988I$	$-8.00000 + 3.46410I$
$b = 1.61803$		

### III.

$$I_3^u = \langle u^3 - 3u^2 + 2b + 3u - 4, -u^3 + 2u^2 + 2a - 2u + 3, u^4 - 3u^3 + 5u^2 - 6u + 4 \rangle$$

(i) **Arc colorings**

$$a_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_1 = \begin{pmatrix} 1 \\ -u^2 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} \frac{1}{2}u^3 - u^2 + u - \frac{3}{2} \\ -\frac{1}{2}u^3 + \frac{3}{2}u^2 - \frac{3}{2}u + 2 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} \frac{1}{2}u^2 - \frac{1}{2}u + \frac{1}{2} \\ -\frac{1}{2}u^3 + \frac{3}{2}u^2 - \frac{3}{2}u + 2 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} \frac{1}{2}u^2 - \frac{1}{2}u + \frac{1}{2} \\ -\frac{3}{2}u^3 + \frac{7}{2}u^2 - \frac{9}{2}u + 4 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -\frac{3}{4}u^3 + \frac{7}{4}u^2 - \frac{9}{4}u + 3 \\ \frac{1}{2}u^3 - \frac{3}{2}u^2 + \frac{5}{2}u - 3 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -\frac{1}{4}u^3 + \frac{1}{4}u^2 + \frac{1}{4}u \\ \frac{1}{2}u^3 - \frac{3}{2}u^2 + \frac{5}{2}u - 3 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -\frac{1}{4}u^3 + \frac{1}{4}u^2 - \frac{3}{4}u + 1 \\ \frac{1}{2}u^3 - \frac{3}{2}u^2 + \frac{3}{2}u - 1 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -\frac{1}{4}u^3 + \frac{1}{4}u^2 - \frac{3}{4}u + 1 \\ \frac{1}{2}u^3 - \frac{3}{2}u^2 + \frac{3}{2}u - 1 \end{pmatrix}$$

(ii) **Obstruction class** = -1

(iii) **Cusp Shapes** =  $2u^3 - 2u^2 + 2u - 10$

**(iv) u-Polynomials at the component**

Crossings	u-Polynomials at each crossing
$c_1$	$u^4 - 3u^3 + 5u^2 - 6u + 4$
$c_2, c_6$	$u^4 + 3u^3 + 5u^2 + 6u + 4$
$c_3, c_5, c_8$ $c_9$	$(u^2 - u + 1)^2$
$c_4, c_7$	$(u^2 + u - 1)^2$

**(v) Riley Polynomials at the component**

Crossings	Riley Polynomials at each crossing
$c_1, c_2, c_6$	$y^4 + y^3 - 3y^2 + 4y + 16$
$c_3, c_5, c_8$ $c_9$	$(y^2 + y + 1)^2$
$c_4, c_7$	$(y^2 - 3y + 1)^2$

**(vi) Complex Volumes and Cusp Shapes**

Solutions to $I_3^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 1.30902 + 0.53523I$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	
$a = -1.059020 + 0.433013I$	$-3.94784 - 2.02988I$	$-8.00000 + 3.46410I$
$b = 1.61803$		
$u = 1.30902 - 0.53523I$		
$a = -1.059020 - 0.433013I$	$-3.94784 + 2.02988I$	$-8.00000 - 3.46410I$
$b = 1.61803$		
$u = 0.19098 + 1.40126I$		
$a = 0.059017 - 0.433013I$	$3.94784 + 2.02988I$	$-8.00000 - 3.46410I$
$b = -0.618034$		
$u = 0.19098 - 1.40126I$		
$a = 0.059017 + 0.433013I$	$3.94784 - 2.02988I$	$-8.00000 + 3.46410I$
$b = -0.618034$		

$$\text{IV. } I_4^u = \langle b^2 - bu + b + 2, a - 1, u^2 + u - 1 \rangle$$

(i) Arc colorings

$$a_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_1 = \begin{pmatrix} 1 \\ u - 1 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} 1 \\ b \end{pmatrix}$$

$$a_4 = \begin{pmatrix} b + 1 \\ b \end{pmatrix}$$

$$a_3 = \begin{pmatrix} b + 1 \\ bu + u - 1 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} u \\ bu + u \end{pmatrix}$$

$$a_6 = \begin{pmatrix} bu + 2u \\ bu + u \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -b + u \\ u + 1 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -b + u \\ u + 1 \end{pmatrix}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes =  $4bu + 4u - 10$

**(iv) u-Polynomials at the component**

Crossings	u-Polynomials at each crossing
$c_1, c_7$	$(u^2 + u - 1)^2$
$c_2, c_3, c_6$ $c_8$	$(u^2 - u + 1)^2$
$c_4$	$u^4 - 3u^3 + 5u^2 - 6u + 4$
$c_5, c_9$	$u^4 + 3u^3 + 5u^2 + 6u + 4$

**(v) Riley Polynomials at the component**

Crossings	Riley Polynomials at each crossing
$c_1, c_7$	$(y^2 - 3y + 1)^2$
$c_2, c_3, c_6$ $c_8$	$(y^2 + y + 1)^2$
$c_4, c_5, c_9$	$y^4 + y^3 - 3y^2 + 4y + 16$

**(vi) Complex Volumes and Cusp Shapes**

Solutions to $I_4^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.618034$		
$a = 1.00000$	$3.94784 - 2.02988I$	$-8.00000 + 3.46410I$
$b = -0.19098 + 1.40126I$		
$u = -1.61803$		
$a = 1.00000$	$-3.94784 + 2.02988I$	$-8.00000 - 3.46410I$
$b = -1.30902 + 0.53523I$		
$u = -1.61803$		
$a = 1.00000$	$-3.94784 - 2.02988I$	$-8.00000 + 3.46410I$
$b = -1.30902 - 0.53523I$		

$$\mathbf{V} \cdot I_5^u = \langle b + u, a + 1, u^3 + u^2 + 1 \rangle$$

(i) Arc colorings

$$a_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_1 = \begin{pmatrix} 1 \\ -u^2 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -1 \\ -u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} -u - 1 \\ -u \end{pmatrix}$$

$$a_3 = \begin{pmatrix} -u - 1 \\ -u - 1 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} u \\ u^2 + u \end{pmatrix}$$

$$a_6 = \begin{pmatrix} u^2 + 2u \\ u^2 + u \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -u^2 - u + 1 \\ -u^2 + 1 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -u^2 - u + 1 \\ -u^2 + 1 \end{pmatrix}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes =  $-3u^2 - 3$

**(iv) u-Polynomials at the component**

Crossings	u-Polynomials at each crossing
$c_1, c_4, c_7$	$u^3 + u^2 + 1$
$c_2, c_5, c_8$	$u^3 + u - 1$
$c_3, c_6, c_9$	$u^3 + u + 1$

**(v) Riley Polynomials at the component**

Crossings	Riley Polynomials at each crossing
$c_1, c_4, c_7$	$y^3 - y^2 - 2y - 1$
$c_2, c_3, c_5$ $c_6, c_8, c_9$	$y^3 + 2y^2 + y - 1$

**(vi) Complex Volumes and Cusp Shapes**

Solutions to $I_3^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.232786 + 0.792552I$		
$a = -1.00000$	$5.50124 + 1.58317I$	$-1.27815 - 1.10697I$
$b = -0.232786 - 0.792552I$		
$u = 0.232786 - 0.792552I$		
$a = -1.00000$	$5.50124 - 1.58317I$	$-1.27815 + 1.10697I$
$b = -0.232786 + 0.792552I$		
$u = -1.46557$		
$a = -1.00000$	$-4.42273$	$-9.44370$
$b = 1.46557$		

## VI. u-Polynomials

Crossings	u-Polynomials at each crossing
$c_1, c_4, c_7$	$((u^2 + u - 1)^4)(u^3 - 3u^2 + 2u + 1)(u^3 + u^2 + 1)(u^4 - 3u^3 + \cdots - 6u + 4)$
$c_2, c_5, c_8$	$((u^2 - u + 1)^4)(u^3 + u - 1)(u^3 + 2u^2 + 3u + 1)(u^4 + 3u^3 + \cdots + 6u + 4)$
$c_3, c_6, c_9$	$((u^2 - u + 1)^4)(u^3 + u + 1)(u^3 + 2u^2 + 3u + 1)(u^4 + 3u^3 + \cdots + 6u + 4)$

## VII. Riley Polynomials

Crossings	Riley Polynomials at each crossing
$c_1, c_4, c_7$	$(y^2 - 3y + 1)^4(y^3 - 5y^2 + 10y - 1)(y^3 - y^2 - 2y - 1) \\ \cdot (y^4 + y^3 - 3y^2 + 4y + 16)$
$c_2, c_3, c_5$ $c_6, c_8, c_9$	$(y^2 + y + 1)^4(y^3 + 2y^2 + y - 1)(y^3 + 2y^2 + 5y - 1) \\ \cdot (y^4 + y^3 - 3y^2 + 4y + 16)$